CONVEXITY

The recent gains in the preferred share market have brought many issues into the range in which convexity of PerpetualDiscount preferreds becomes important, so this short essay will review the concept.

Convexity is a mathematical term used in fixed income analysis to describe the second derivative of the equation:

$$Y = f(P) \quad (1)$$

Where: Y = yield
P = price
f( ) is the function relating the two attributes

This isn't as scary as it looks. The first derivative of this equation is Modified Duration, which I have discussed in an essay available via http://www.prefblog.com/?p=864; Modified Duration describes the first-order approximation of the relationship between the yield of a fixed income instrument and its price; for example, a five year bond will have a modified duration of slightly less than its term, about 4.5. This is interpreted as meaning that a change of 1% in the absolute yield of a bond (say, from 4% to 5%) will change its price (approximately) according to the formula

$$\Delta P/P = -D_{MOD} \bullet \Delta Y \quad (2)$$

Where: ΔP = change in price
P = price
D_{MOD} = Modified Duration
ΔY = change in yield

Since the Modified Duration of a five year bond is about 4.5 (the number will vary, depending upon the coupon rate, the yield at which the issue is trading and the coupon frequency), we may conclude that a change in yield from 4% to 5% will result in a capital loss of approximately 4.5%.

Alert readers will have noted that I have used the word approximately several times while explaining the meaning of Modified Duration – this is because equation (2) is only a first order approximation to the true relationship between price and yield, even for normal, non-callable, bonds. A better description of the relationship between price and yield is

$$\Delta P/P = -D_{MOD} \bullet \Delta Y + ½ \bullet C \bullet \Delta Y^2 \quad (3)$$

Where: C = Convexity
Other symbols as defined in equation (2)

The calculation of Convexity for normal bonds has been discussed in a previous essay accessible via http://www.prefblog.com/?p=1640.

For all normal bonds convexity is positive, but when we bring embedded options into the mix – as we do when considering preferred shares of all descriptions – convexity can become not just negative, but sometimes large and negative, which describes a situation in which percentage capital losses accelerate markedly with increasing yield. Positive convexity works in favour of the investor; negative convexity works against the investor.

Fortunately, the negative convexity effects on preferred shares due to their embedded options is not as drastic as it could be with other types of options (futures contracts on bonds, for instance, allow the seller to determine which of several different bonds to deliver in order to settle his obligations; this option can imply a very large negative convexity depending on the basket of deliverable bonds and the changes in yields considered), but must never-the-less be examined.

All perpetual preferred shares have an embedded option: the issuer has the right, but not the obligation, to call the issue at par at a given time after the issue date (most also have the ability to call prior to this time, at a premium to par). Since it is the issuer's decision whether or not to exercise this option, one may be sure that the feature makes the investment worse for the investor (better for the issuer) than if the feature was absent and the issue was a true perpetual.

In this discussion, I shall treat the two elements of a PerpetualDiscount preferred share separately. They are:

- A true (non-callable) perpetual (easy to price), and
- A short call option on the perpetual (somewhat more difficult!)

The Black-Scholes Option Pricing Equation

Options may be priced according to the Black-Scholes model:

$$OP = SN(d_1) - Xe^{-rt}N(d_2) \quad (4)$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{v^2}{2}\right)}{v\sqrt{t}} \quad (5)$$

$$d_2 = d_1 - v\sqrt{t} \quad (6)$$

Where: S = stock price
X = strike price
t = time remaining until expiration (years)
r = current continuously compounded risk-free interest rate; or carry for income-paying instruments
v = annual volatility of stock price
ln = natural logarithm
N(x) = standard normal cumulative distribution function
e = the exponential function

It should be noted that:

- This is the formula for a European option, which is exercisable only at the expiration date, as opposed to an American option, which may be exercised at any time
- The stock is assumed to pay no dividends
- The effect of dividends may be incorporated in the formula by redefining "*r*" as the cost of carry; that is, through the assumption that the issuer may:
  - Set aside funds that will cover the cost of option exercise
  - Invest these funds at a given rate
  - Pay the dividends on the preferred shares (for the purpose of this part of the calculation tax effects are ignored)
- N(x) is a Gaussian distribution (calculable on Excel via the function NORMSDIST())

Parameterizing the Equation

In order to prepare the charts in this essay, I have made some assumptions and adjustments to the normal formula. Firstly, I used yield volatility rather than price volatility, assuming that the expected yield of a "pure" perpetual is 6.3%, with a standard deviation of this yield of 0.5542% (absolute, not relative to the base yield) annually. I also assumed that the exercise date of this option was three years hence (which sets the variable "*t*" required to solve equations (5) and (6)), so the projected yield on exercise date is 6.3% with a standard deviation of 0.96% (absolute). This assumption results in the probability distribution shown in Chart 1 (which shows the probability for each dividend interval) and Chart 2 (which shows the cumulative probability that the yield will be below the given value of x).

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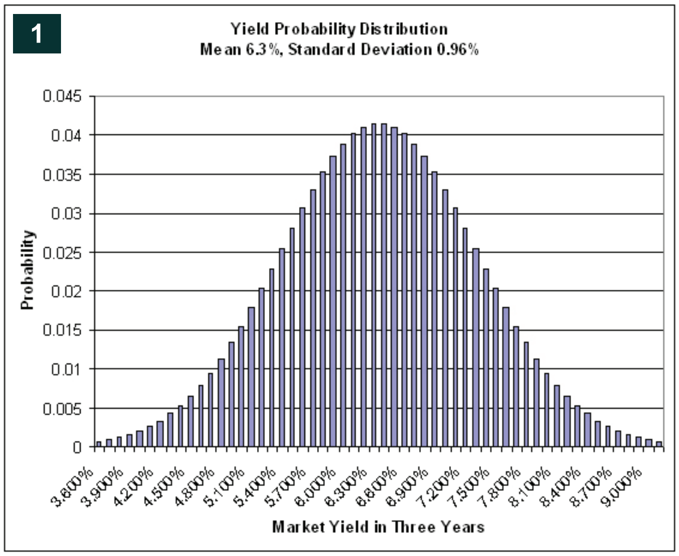

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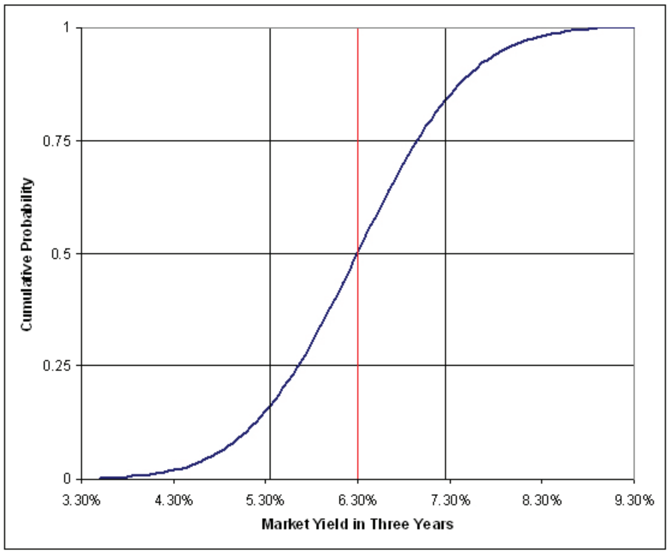

In Chart 2, vertical lines denoting the expected value and the boundaries of one standard deviation from expectation have been provided.

As noted, the volatilities used in the calculation have been assumed, not calculated. Quantitative Analysis does not consist of slavishly inputting historical data regardless of the broad economic factors that may have influenced the movements of the data; Andrew Haldane of the Bank of England pointed out the nonsensical nature of the pseudo-quantitative statement that "We are seeing things that were 25-standard deviation moves, several days in a row" in a speech last February (see http://www.prefblog.com/?p=5556). Similarly, the statement that "California has never failed to [pay you your money on time and in full]" (see http://www.prefblog.com/?p=2955) may well be both accurate and interesting, but is not particularly useful when estimating California's current risk of default. Quantitative Analysis, performed properly, consists of the rigorous application of theory to market data, using parameters – when necessary – that are not only robust to the data, but may be expected to remain robust in the future.

Thus, having selected a volatility that seems reasonable, we perform a sanity check through via available data. Chart 3 shows the relative (not absolute) change in yields of the PerpetualDiscount index over three year terms for the period covered by the HIMIPref™ database: commencing 1993-12-31, the median-by-weight YTW of the index was measured and compared with a similar figure three years later. If either figure was not available (there have been long periods during which there were simply no PerpetualDiscounts trading in the marketplace; the last one extended from November 30 to December 30, 2005), the data point was discarded.

In order to minimize the effect of changing overall yields, the change in yield was recorded on a relative basis, that is:

$$\Delta = (Y_2 - Y_1) / Y_1$$

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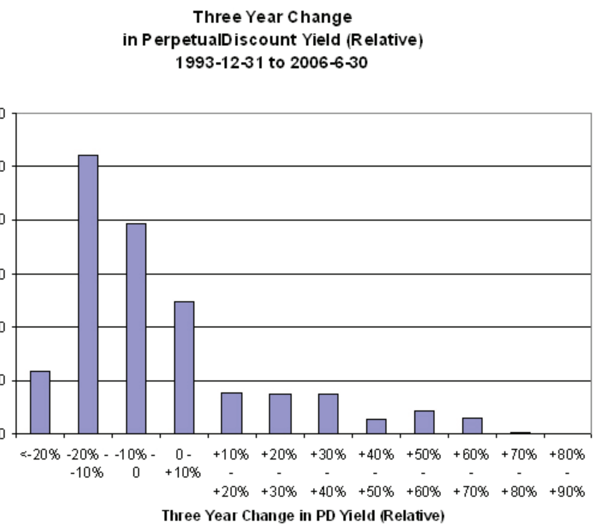

Where: Δ is the relative change in yield
Y_1 is the initial yield
Y_2 is the yield three years afterwards

When we examine the 1602 valid data points, we may prepare Chart 3; it may be seen that there is a large positive skew to the data (thus the Gaussian Distribution assumed by the model is rendered somewhat suspect), but that the estimate of a 0.96% absolute three-year standard deviation in the projected future yield of 6.3%, which is a relative standard deviation of 0.96%/6.3% = 15%, is well within the bounds of what may be considered reasonable.

Having arrived at an estimate of the volatility required for equations (5) and (6), we may turn our attention to the cost of carry. The need to determine this value is illustrated by the consideration of two portfolios:

Portfolio A: One (European) call option plus cash sufficient to exercise this option, invested at the risk-free rate until option expiry

Portfolio B: One non-dividend paying share

The lower bound for the price of the call is determined by setting the portfolios to have equal value; if the price was lower then all investors would prefer Portfolio A and arbitrage would move the values back to equivalency.

For analytical convenience, discussions of the Black-Scholes Model usually proceed directly from this assumption, but in the case of preferred shares the dividend paid in the period between acquisition and potential call is not simply important, it is vital. In order to preserve the relationship between Portfolios A and B, we must add cash to Portfolio A such that the amount is sufficient to:

(i)exercise the option at expiry, and
(ii)pay the dividend on the share (which is equivalent to receiving the dividend in Portfolio B)

This is achieved by replacing the risk-free rate usually used in the model with the cost of carry. For purposes of the calculations, I have assumed that the issuer (who owns the call) can set aside the funds required for exercise and invest these funds at 2% – this is reasonably close to the yield on three year Canada bonds, which has been at approximately 1.8% lately.

As for the dividend payments required, the first set of calculations has been performed on a notional perpetual preferred share paying a dividend of $1.25 on its par value of $25; a yield at issuance of 5% which will be used as the "Strike Yield" (equivalent to X in equation (4)).

Simple summation of the two figures results in a cost of carry of -3%, and we have therefore defined the variable "r" required for equations (4) and (5).

Finally, we will assume that the yield of "Pure" Perpetual (that is to say, one that is not subject to calls and will pay its dividend forever, barring bankruptcy) is currently 6.3%, equal to the mean projected yield on the exercise date and reasonably close to the median average YTM on the PerpetualDiscount index today. Since we are using yields, rather than prices to solve equation (4), we need not make any assumptions or calculations regarding prices of individual issues.

Effect of Varying Market Yields

All the variables required for the solution of the Black-Scholes equation (given here as Equation (4)) have now been defined. Note that since we are using yields and their volatility, we must invert the market yield, required for variable "S"; so that an actual market yield of 6.3%, which is 1.3% (absolute) above the presumed 5.0% strike yield, is inverted for calculation purposes to 3.7%, which is 1.3% (absolute) below the strike yield.

The initial results of these calculations are illustrated in Chart 4, which shows the value of the option (expressed as an absolute yield differential) vs. a varying market yield. At the current market yield of 6.3%, the value of the option is approximately 0.03%, or 3 basis points (bp), when expressed in terms of yield. If the market was trading at a yield of 5.00%, equal to the strike yield, the fair-value yield of the notional preferred share with an annual dividend of $1.25 would be 5.34%, comprised of 5.00% for the pure perpetual and 34bp for the option.

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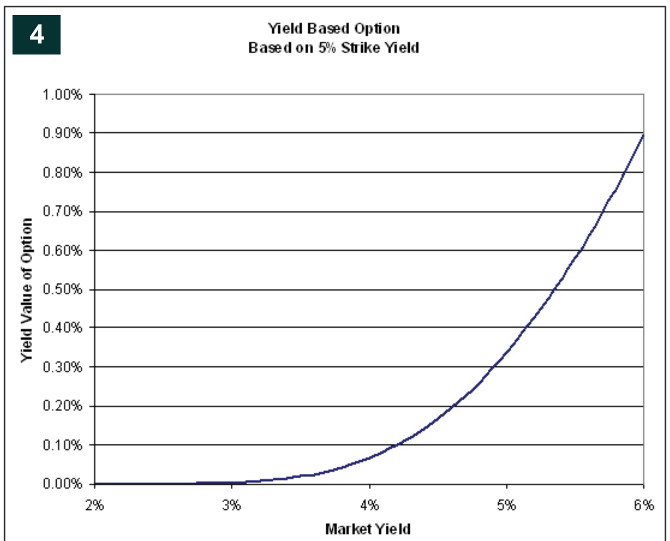
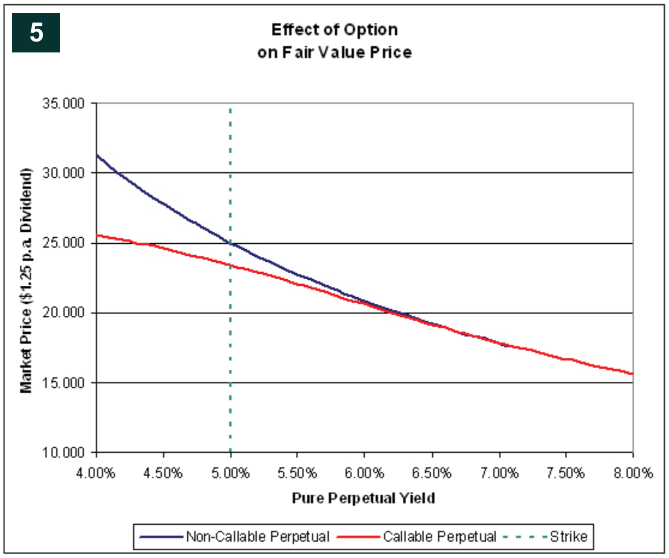
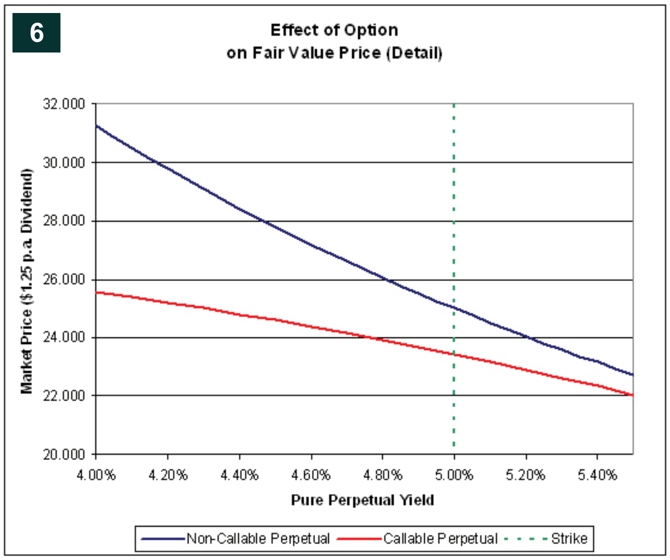
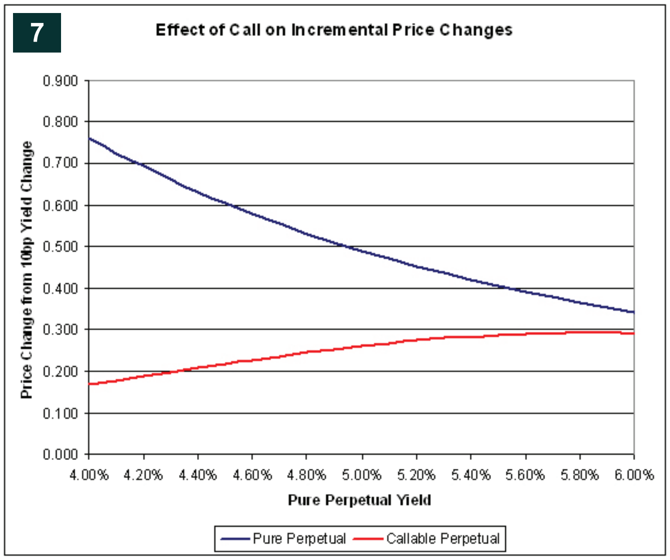






The effect of the options on the price of the preferreds is illustrated in Charts 5 and 6. The latter chart is simply a detail view of part of the former, making it easier to see the relative curvature of the "Pure Perpetual" and "Callable" prices. It may be seen that Pure Perpetual relationship is slightly curved so that the slope increases (that is, becomes less negative) with increases in the value of Market Yield (Positive Convexity) while the slope of the Callable relationship decreases (becomes more negative) with increasing Market Yield (Negative Convexity).

Finally, for this series of calculations based on the notional $1.25 p.a. perpetual, Chart 7 shows the incremental price changes resulting from a change in yields. This shows the difference between the positive and negative convexity of the two notional instruments more clearly.



The Effect of Varying Annual Dividends

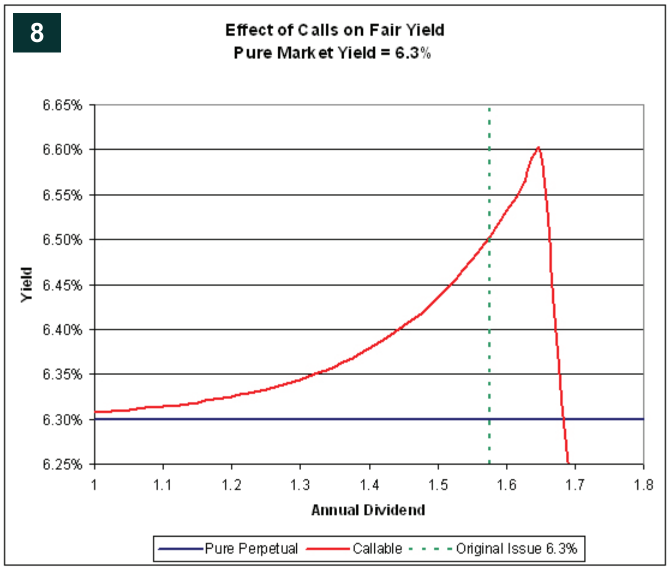
Thus far we have looked at the effect of changing market yields on the value of the embedded option; for practical purposes it is more useful to look at how differing dividend rates relate to a constant market yield so that we may assign a fair price to instruments carrying these dividends.

To prepare Chart 8, I have assumed that the fair value of a Pure Perpetual is 6.3% – whatever the annual dividend might be, the price will be determined so that the annual yield will be 6.3%. When we look at the effect of calls, however, assuming that all instruments may be called at their original issue price of $25.00, we see some dramatic effects.

As the annual dividend increases, the value of the option sold reaches 30bp when the annual dividend is $1.65. A Pure Perpetual would be trading at the pure yield of 6.3%, at a price of $26.19, but the option inherent in the Callable Perpetual means that the fair value yield of such an issue is 6.6% and the issue will trade at only $25.00.

If we look at comparable issues that pay an even higher dividend, the presumption of a $25.00 call price means that the option has become "in the money" … a Pure Perpetual with a dividend of $1.90 would trade at $30.16 to achieve the 6.3% market yield, but the Callable issue is hit by two factors:

- The option has a yield value of 0.93%, implying a fair-value perpetual yield of 7.23% and a price of $26.29
- The yield-to-worst scenario has become the exercise of the call at $25.00 in three years; the yield in this scenario (which accounts for the resultant capital loss of $1.29) is a mere 5.74%



Implications for New Issues

The practical implications of the above discussion should be clear: if we look at, for instance, SLF.PR.E we note that it pays a dividend of $1.125 p.a., and was priced at the close on July 9 at $16.85-94, deeply discounted from its call price of $25.00. The Current Yield of this issue at the bid price is therefore 6.68% (I shouldn't use Current Yield, I know, but this is only a quick example. The properly calculated yield, which accounts for the proximity of the next ex-Dividend date and converts the yield to semi-annual compounding, is 6.74% … and if you think the difference is inconsequential, please call me next time you want to execute a trade!).

Now suppose Sun Life comes out with a new issue of another Straight Perpetual, paying a dividend of $1.67 p.a. and priced at $25.00 to show a Current Yield of 6.68%. Should an investor swap his SLF.PR.E for the new issue?

The alert reader will immediately snarl his response: "No!". The embedded call on the new issue is worth 30bp, while the embedded call on SLF.PR.E is close to worthless. Executing the swap means letting Sun Life have an option worth 30bp yield for free; not usually considered the formula for investment success. In order for the investor to become indifferent between SLF.PR.E and the new issue, the latter must yield 30bp more, 6.98%, paying a dividend of $1.745.

And this extra dividend is required merely to become indifferent. If Sun Life wishes the investor to perform a complicated operation like making a telephone call; paying commission on his sale of SLF.PR.E and possibly becoming liable for Capital Gains Taxes … there must be more dividend beyond the $1.745 indifference level, representing Sun Life's new issue concession.

Current Market Pricing of Embedded Options

On a day-to-day basis, we are interested not so much in the implications for the pricing of new issues; we are attempting to see which of the available issues on the secondary market are mispriced. If we are able to judge the mispricing accurately – and if we are able to trade sufficiently cheaply to take advantage of these mispricings – then, eventually, our portfolios will outperform.

Accordingly, we'll have a look at the Current Yields of four issuers; each of these issuers has a number of PerpetualDiscounts outstanding and the spread between the high and low dividend for each of these issuers is equal to or greater than $0.30 p.a.

These issuers are BMO, CM, RY and GWO and the data analysis is shown in Chart 9.

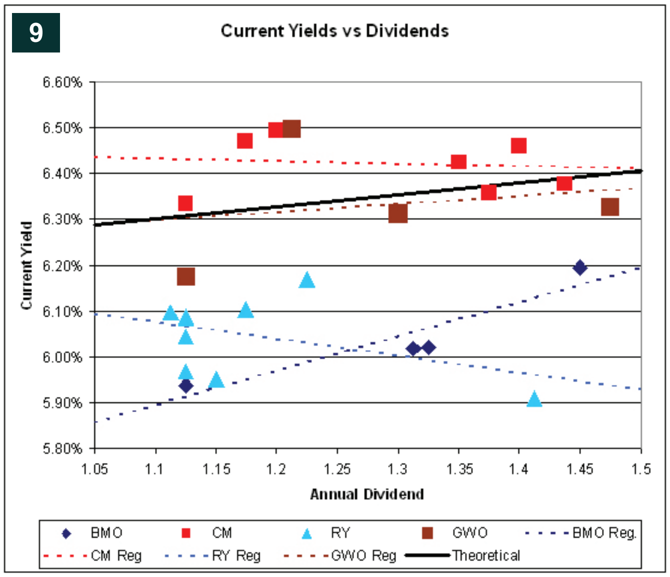


Table 1 shows the results of regressing the Current Yields of the outstanding PerpetualDiscounts by Issuer:

Table 1: Results of Regressing Current Yields Against Dividends

Issuer	Intercept	Slope
BMO	5.07%	+0.0075
CM	6.49%	-0.00054
RY	6.48%	-0.00368
GWO	6.10%	+0.00179
Theoretical	6.02%	+0.002571

The intercepts shown in Table 1 are without much meaning; if the regression were to be interpreted strictly, it would represent the Current Yield of an issue paying zero dividends. Clearly, we must specify that the relationship found by the regressions is valid, at best, in the interval between annual dividend payments of $1.05 and $1.50!

It is the slope that is most important, and we see from the entry for the "Theoretical" model (taken from the data used to prepare Chart 8; performing the regression for data points representing dividends within the valid range) that the slope should be positive; that is, that the fair-value yield should increase with increasing dividend which has been the central point of this essay.

In fact, none of the regressions of the market data provides a slope that excludes "zero" from its 95% confidence interval; as is so often the case with the preferred share market, there is simply not enough data to allow for the isolation of a single explanatory variable that holds all other influences (such as credit quality, liquidity, etc.) constant.

It should also be noted that market sometimes behaves in such a way as to defy rational explanation: in June of 2008, for instance, the market briefly assigned the options a negative value (a negative slope, in terms of the table; negative, large and meaningful when pricing effects are examined), as discussed in my essay *The Swoon in June* available on-line via http://www.prefblog.com/?p=3126. I remember this episode with particular clarity since my fund, Malachite Aggressive Preferred Fund (MAPF; see http://www.himivest.com/malachite/MAPFMain.php) performed very poorly that month; the notorious ability of the market to remain irrational for long periods has brought many leveraged investors to grief – fortunately MAPF does not employ leverage as an investment stategy!

However, the very lack of agreement of market pricing (as it existed on July 10, 2009, at about 4pm, anyway!) with theory may be used by investor to indicate potential rich/cheap relationships. Consider, for example, a comparison between RY.PR.F and RY.PR.H, as shown in Table 2:

Table 2: Comparison of RY.PR.F & RY.PR.H

Issue	Dividend	Price	Current Yield
RY.PR.F	1.1125	18.25	6.10%
RY.PR.H	1.4125	23.90	5.91%

Note that it is permissible to use Current Yield for comparative purposes since the ex-Dividend dates are the same and payment frequencies are identical; were we to compare issues with different ex-Dividend dates, we would have to use the more precise YTW calculation. It should also be noted that there is no difference in the credit quality of these issues.

Why would anybody invest in RY.PR.H, given this comparison? Not only does the issue yield less, but it is also giving up a more expensive option. Clearly, RY.PR.F is the better choice of the two alternatives, although no claims should be inferred regarding its investment quality versus any of the other possibilities!