Implied Volatility for FixedResets

In this essay I explain the calculation of Implied Volatility for FixedResets, in accordance with a spreadsheet I have developed which is publicly available at http://www.prefblog.com/xls/ImpliedVolatility.xls. The spreadsheet is compatible with MS-Excel 2003, and must be downloaded to your hard drive in order to work.

The Black-Scholes Option Model

In order to price the embedded call, we may use the Black-Scholes option pricing model, a complex mathematical formula based on sometimes dubious assumptions. A good overview of the technique is available at http://bradley.bradley.edu/~arr/bsm/model.html.

Briefly, the Black-Scholes model assumes that the best estimate of a financial instrument's future price is its current price, but that the actual future price will vary around this estimate in a well-defined way – a bell curve. The price of an option will be determined by the chance that it will be valuable at the time of its expiry: if you have an option with a 10% chance of being worth a dollar and a 90% chance of being worthless; the option's price should be ten cents (ignoring adjustments for the time value of money).

The width of the bell curve of possible future prices is dependent upon two factors: the time to expiry of the option (naturally, the longer to expiry, the wider the distribution) and the volatility – a measure of how much change in the price may be expected in a standard period of time (usually a year).

Volatilities do not have to be expressed in terms of price; it is entirely admissible to perform the calculation in terms of yields, which then provides results in terms of yields.

Additionally, it may be shown[1] that: *The interpretation of N(d1) is a bit more complicated. The expected value, computed using risk-adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is N(d1) multiplied by the current stock price and the riskless compounding factor. Thus, N(d1) is the factor by which the present value of contingent receipt of the stock exceeds the current stock price.*

The present value of contingent receipt of the stock is not equal to but larger than the current stock price multiplied by N(d2), the risk-adjusted probability of exercise. The reason for this is that the event of exercise is not independent of the future stock price. If exercise were completely random and unrelated to the stock price, then indeed the present value of contingent receipt of the stock would be the current stock price multiplied by N(d2). Actually the present value is larger than this, since exercise is dependent on the future stock price and indeed happens when the stock price is high.

Note, however, that the calculation of N(d2) and N(d1) in the spreadsheet are based on the Issue Reset Spreads, not on price.

Data and Calculations Performed in the Spreadsheet

On the spreadsheet note that green cells contain semi-permanent data, updated infrequently; yellow cells contain fitting data, updated by the user whenever an observation is made; and purple cells contain calculations.(and, in the case of column B only, external data).

Ticker: Column A, semi-permanent data: This ticker symbol takes the format specified by Yahoo! so that bid prices for all instruments may be easily loaded.

New Bid: Column B, external data: This is the most recent bid price on the Toronto Stock Exchange, as downloaded on the tab labeled "Yahoo".

Spread: Column C, semi-permanent data: This is the Issue Reset Spread, expressed in basis points.

Annual Dividend: Column D, calculated data: This is the dividend rate, in dollars, to which the issue will reset on the next exchange date, given the Spread specified in Column C and the GOC-5 rate specified by the user.

Expected Current Yield: Column E, calculated data. This is the Current Yield of the instrument, given the Annual Dividend calculated in Column D and the Price provided in Column B. It will be noted that this approximation introduces an error into the calculation – ideally, the actual current dividend would be provided as semi-permanent data and the difference between this value and the Annual Dividend calculated in Column D would be multiplied by the time until the actual Reset Date (which would also have to be provided). This difference between the dividends received according to the calculation and the dividends legitimately expected to be received would be used to adjust the bid price – possibly with an adjustment for the Present Value of the difference, rather than the undiscounted total. After consideration, I assumed that the error involved in the approximation used would be less than normal market noise, but I have not checked this assumption.

[1] Lars Tyge Nielsen, *Understanding N(d1) and N(d2): Risk-Adjusted Probabilities in the Black-Scholes Model*, available on-line at http://www.ltnielsen.com/wp-content/uploads/Understanding.pdf (accessed 2013-9-7)

Pure Price: Column G, calculated data. This is the value of an uncallable Perpetual Annuity paying the annual dividend, discounted at a rate equal to the sum of the user inputs GOC-5 and Market Spread.

Option Adjustment: Column H, calculated data. This is simply the call premium calculated in Column Q. It is copied here for user convenience in reading the overall results of the calculation in columns G through L.

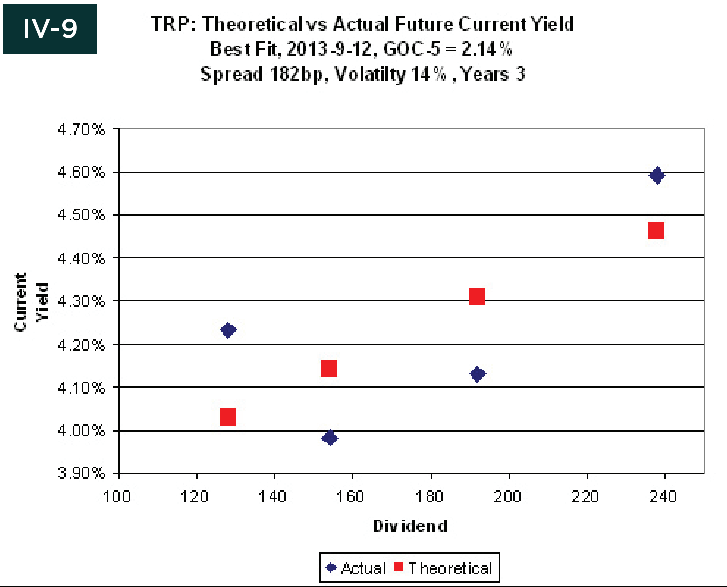
Theoretical Price: Column I, calculated data: This is the sum of the Pure Price (Column G) and the Option Adjustment (Column H). This is the price at which the instrument should be trading given the market data provided by the user and the assumptions discussed in this section.

Squared Error: Column J, calculated data: This is the square of difference between the actual price (Column B) and the Theoretical Price (Column I), which is the error shown in Column L. The user is expected to minimize this error by varying the inputs marked in yellow.

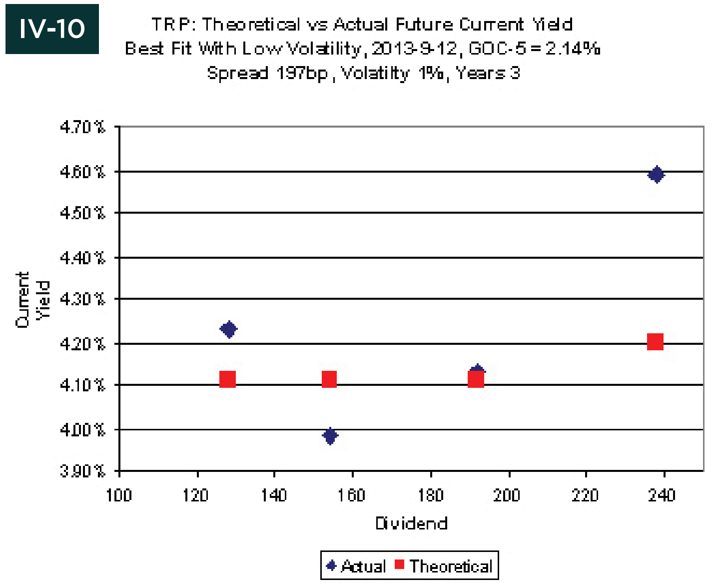
Theoretical Current Yield: Column K, calculated data. This is the Expected Annual Dividend (column D) divided by the Theoretical Price (Column I). It has no major importance, but is very useful for visualizing the relationships in the automatically generated chart.

Error: Column L, calculated data. The raw difference between the Theoretical Price (Column I) and the Actual Price (Column B). It is useful to be able to add the squared-errors of column J separately according to the sign of the raw error; this enables the user to center the theoretical prices by varying spread and then adjusting the slope of the relationship by varying volatility. An example of the effects of varying volatility shown in Charts IV-9, IV-10 and IV-11.

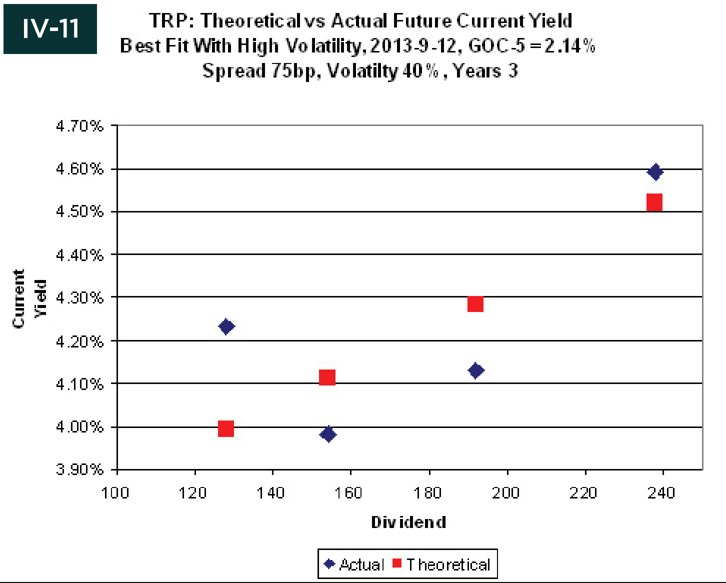
IV-9



IV-10



IV-11



Term 1: Column N, calculated data: This is the first term of the Black-Scholes equation, which, as quoted from Nielsen, above, is: *The expected value, computed using risk-adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is N(d1) multiplied by the current stock price and the riskless compounding factor.* Specifically to this spreadsheet, this is the Pure Price (column G) multiplied by N(d1) (Column AC)).

e^-rt: Column O, calculated data: This is the discounting factor required for Term 2 (Column P) and is the equal to the base of natural logarithms ("e"), to the exponent of the negative of the risk free rate ("r", calculated in Column Z) multiplied by the term to option exercise ("t", from Column X).

Term 2: Column P, calculated data: This is the second term of the Black-Scholes equation, which, as quoted from Nielsen, above, is: *the present value of contingent receipt of the stock … when the stock price is high.* It is equal to e^{-rt} (the discounting factor from column O) multiplied by N(d2), the cumulative normal distribution function calculated in column AD.

Call Premium: Column Q, calculated data. This is the value of the call option, equal to the difference between Term 1 (calculated in column N) and Term 2 (calculated in column P).

T Years: Column X, calculated data copied from user input. This is the term in years until option exercise and is simply copied from the user input data. It will be noted that in this calculation, T is the same for all instruments, which introduces an error into the calculation – since, of course, Exchange Dates in a set of FixedResets from the same issuer will normally be unique for each issue. I believe that this approximation will have little effect on the calculation compared to the normal vagaries of the market. As the market becomes more efficient, this approximation may have to be reviewed!

Sigma: Column Y, calculated data copied from user input. This is the volatility of the Market Reset Spread as input by the user.

RiskFree: Column Z, calculated data. This is the risk-free rate that is of great importance in the Black-Scholes formulation and here again I have made a design decision that some may consider to be less than optimal. The Black-Scholes model sets the Call Premium equal to the cost of delta-hedging exposure to securities; the risk-free rate is the amount paid to finance a long position, or received from the investment of proceeds of a short position.[2] In most expositions of the Black-Scholes theory, it is assumed that the cost of carry is the risk-free rate, since the underlying stock pays no dividends. This simplifying assumption is assuredly not the case with preferred shares, so the risk free rate is adjusted by the dividend yield on the preferred. In this spreadsheet I have assumed that

- The risk-free rate is the five-year GOC rate input by the user, and
- The Expected Current Yield from Column E is the benefit (cost) from being long (short) the stock; note that this ignores tax effects.
- The cost of carry is the difference between the two.

d1: Calculated value, Column AA: Calculated in standard Black-Scholes fashion from the Issue Reset Spread (column C), Market Spread (User input), risk-free rate (Column Z), Sigma (Column Y), and Term (Column Z). Here again is a design decision that some may see fit to criticize. Black-Scholes is normally calculated with stock prices, but in this case the stock price includes the option, which at the very least will make the calculations fearsomely calculated. It might be possible to use the Pure Price calculated in Column G, but in the end I decided to use the Issue Reset Spread (Column C) and the Market Spread (User Input) as the determinants of call probability. Note that this means that

- It is assumed that the Market Spread is log-normally distributed, and therefore
- Market Spread must be positive.

d2: Calculated value, Column AB: Calculated in normal Black-Scholes fashion from d1 (Column AA), Sigma (Column Y) and Term (Column X).

N(d1): Calculated value, Column AC: The Microsoft-Excel NORMSDIST function applied to d1 (Column AA).

N(d2): Calculate value, Column AD: The Microsoft-Excel NORMSDIST function applied to d2 (Column AB).

2 Joel R. Barber, *Delta Hedging with Black-Scholes Model*, available on-line at http://www2.fiu.edu/~barberj/s-chpt15.pdf (accessed 2013-9-12)

The Effect of GOC-5 On Theoretical Prices

All analyses in this section were performed with the following variables held constant:

Market Spread: 190bp
Volatility: 14%
Years: 3

Effect of GOC-5 on Pure Price

The Pure Price is the price of a perpetual annuity paying the indicated dividend, given the market yield.

Since:

Market Yield = GOC-5 + Market Spread (1)

And:

Annual Dividend = (GOC-5 + Issue Spread) * 25 (2)

And:

Pure Price = Annual Dividend / Market Yield (3)

Then substitute (2) into (3):

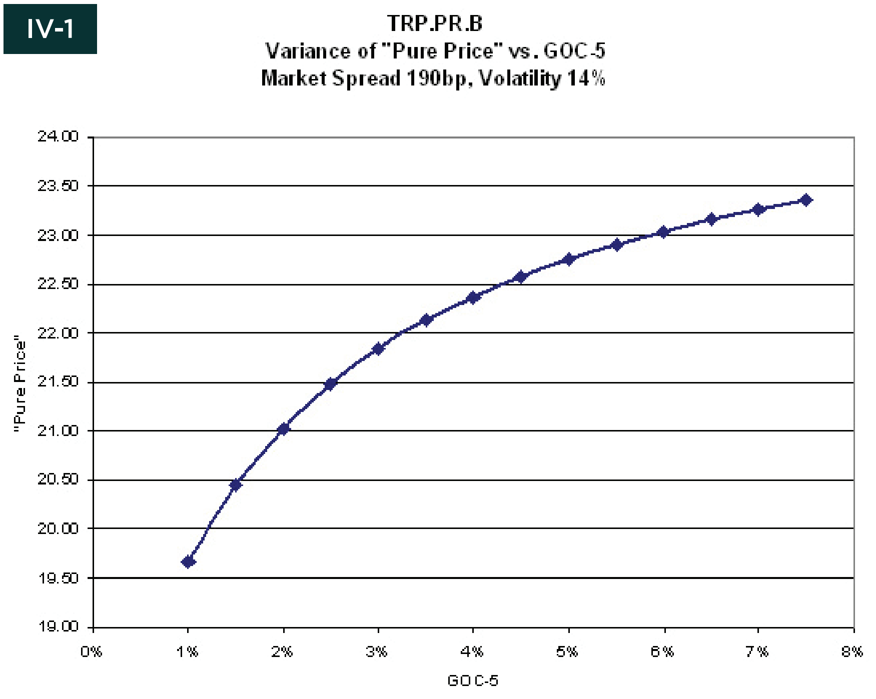
Pure Price = (GOC-5 + Issue Spread) * 25 / Market Yield (4)

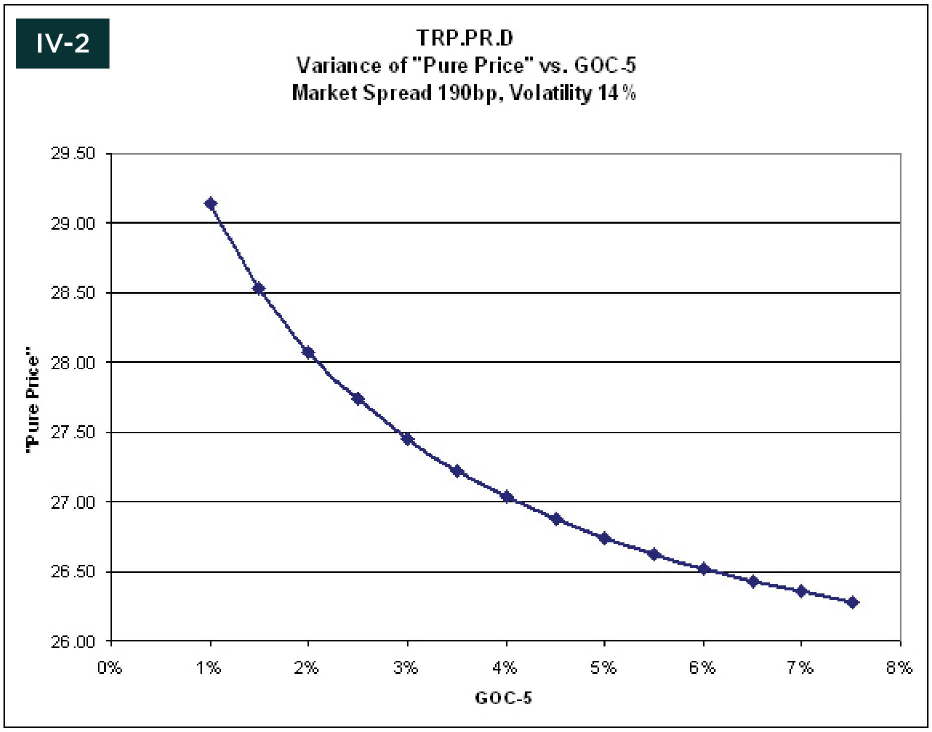
And substitute (1) into (4):

Pure Price = (GOC-5 + Issue Spread) * 25 / (GOC-5 + Market Spread) (5)

Clearly, therefore, as GOC-5 increases (theoretically until Issue Spread and Market Spread become so relatively small that they are effectively zero) the Pure Price will approach 25.00; this is illustrated for TRP.PR.B and TRP.PR.D in Charts IV-1 and IV-2.

IV-1

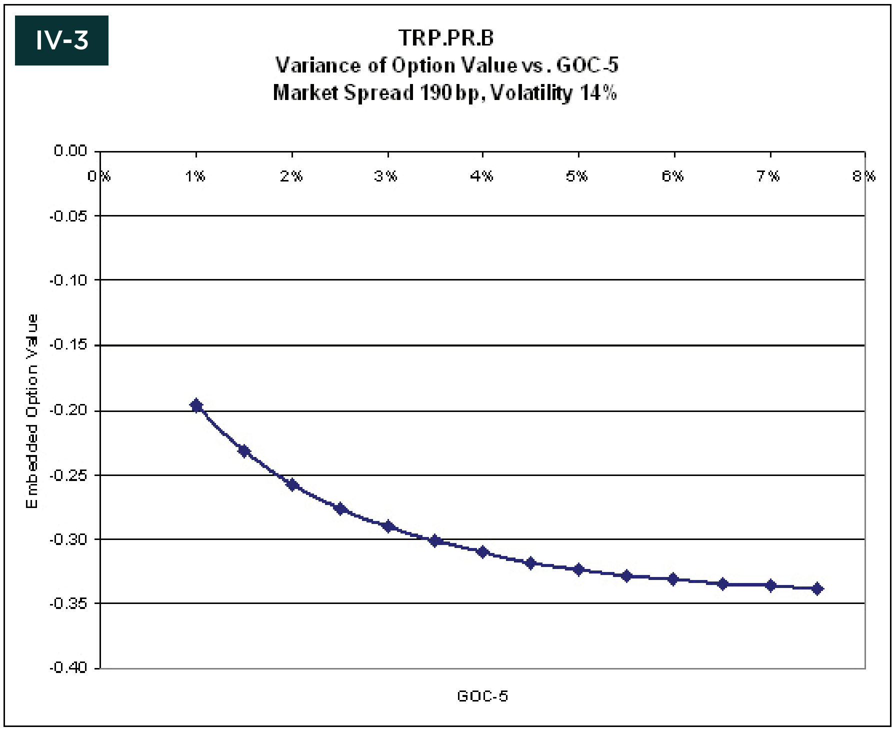


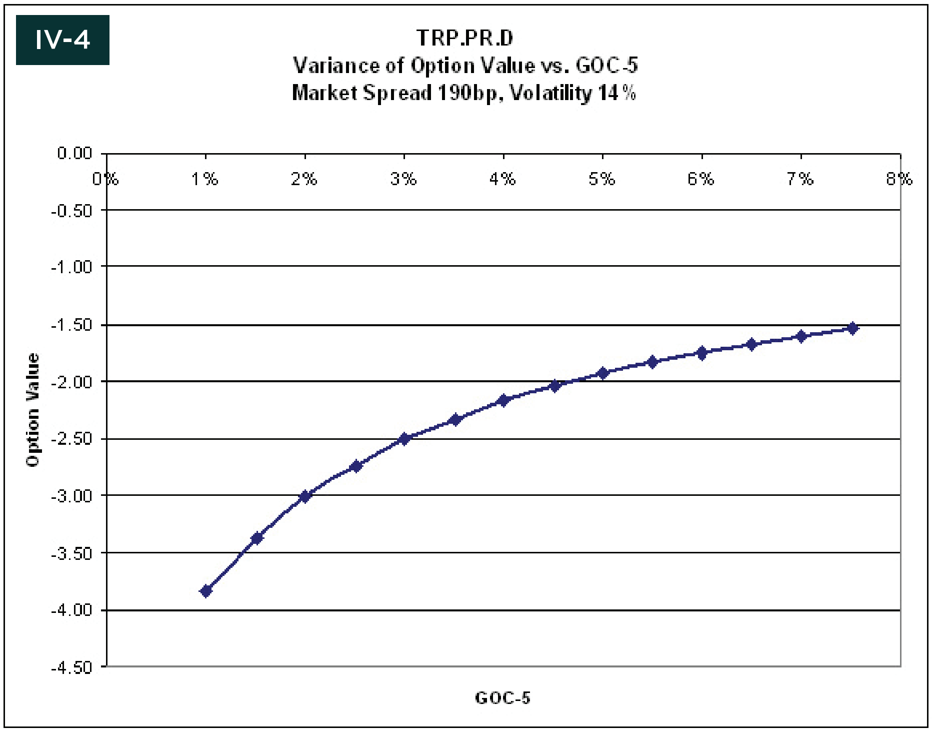


Effect of GOC-5 on Embedded Option Value

Once the Pure Price has been evaluated, we need to include the value of the embedded option in order to determine the Theoretical Price.

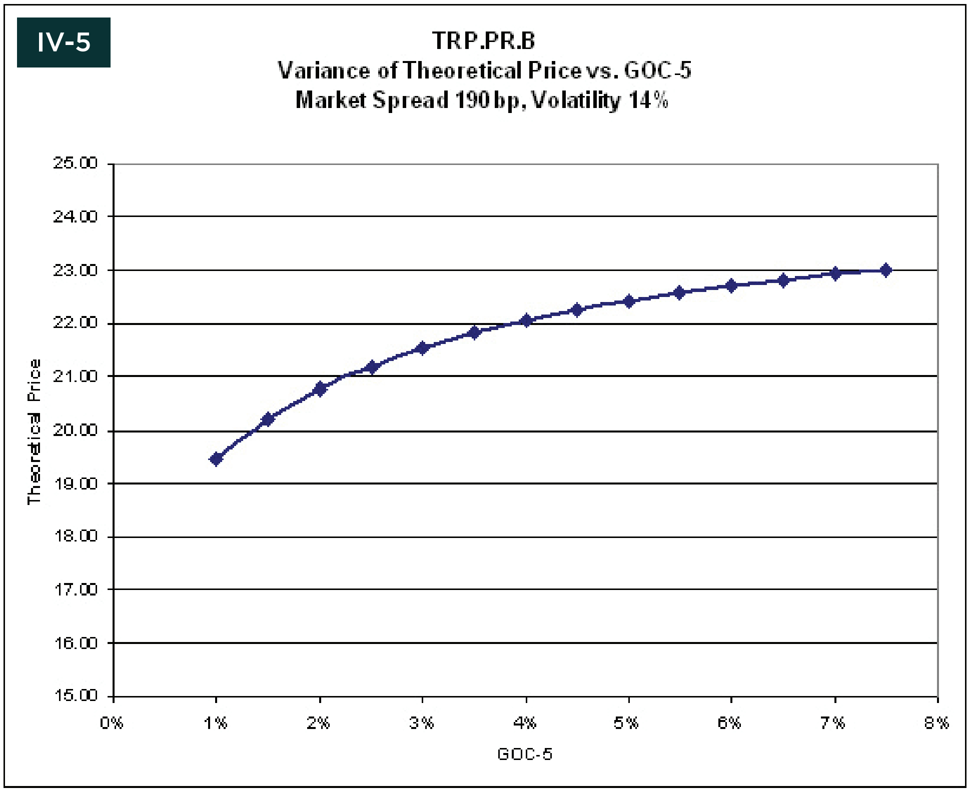
While the details are not especially intuitive, the general trend is clear: since the Pure Price will approach 25.00 as GOC-5 increases, the option value will decline if the Pure Price is above par, and increase if below, as shown in Charts IV-3 and IV-4.



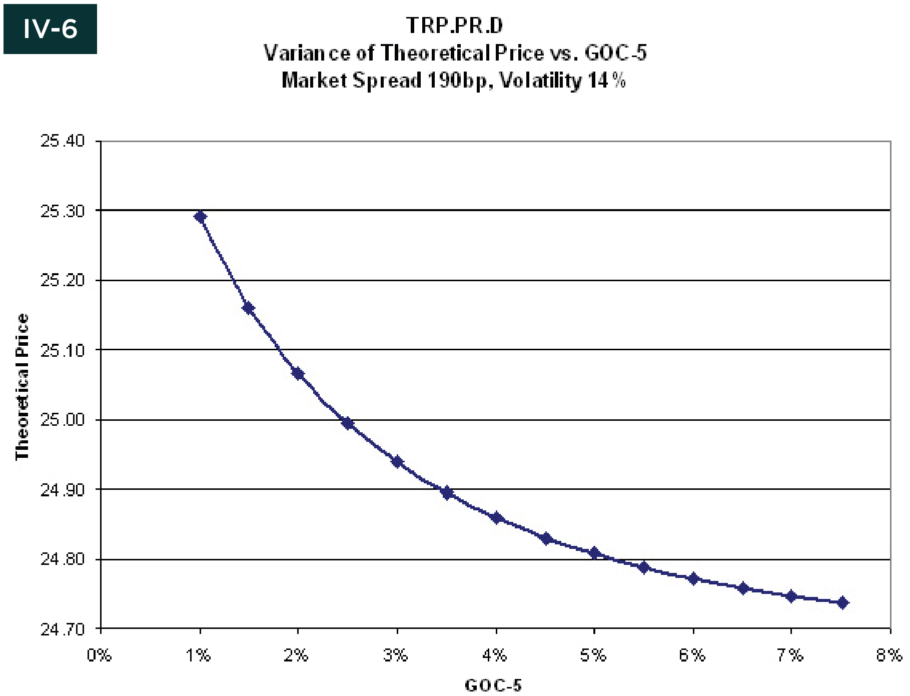


Effect of GOC-5 on Theoretical Price

The theoretical price is the sum of the Pure Price and the embedded option value, which move in opposite directions as GOC-5 increases. The net effect of these trends is shown in Charts IV-5 and IV-6.



IV-6

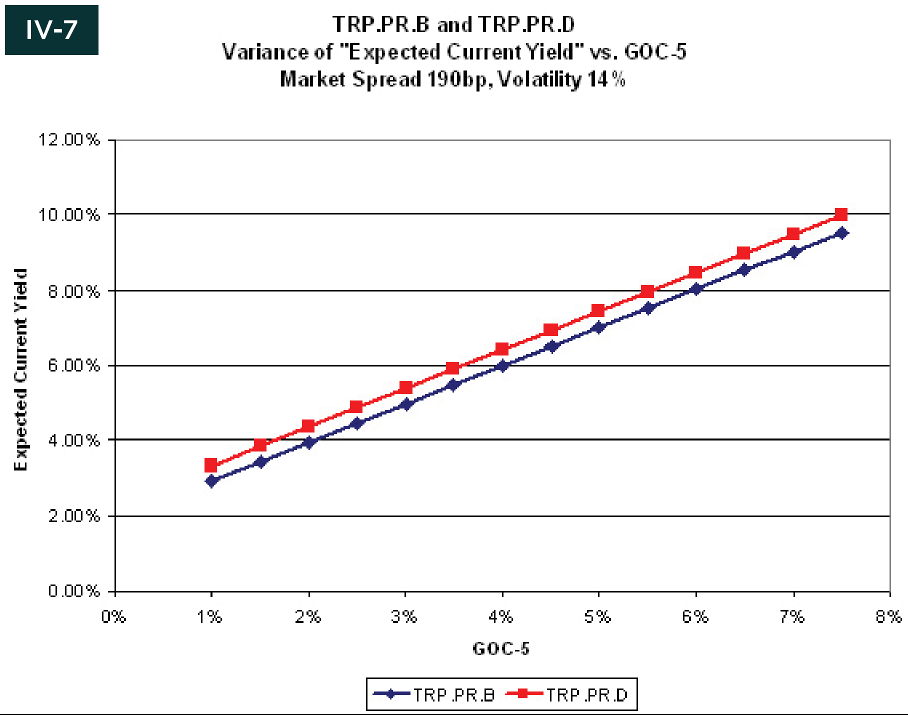


Effect of GOC-5 on "Expected Current Yield"

The "Expected Current Yield" is the dividend expected following reset at the indicated GOC-5 rate, divided by the current theoretical price.

As may be seen from Chart IV-7, these figures rise smoothly with increases in GOC-5; the slope is very slightly greater than 1; that is, an increase of 100bp in the absolute GOC-5 rate may be expected to cause an increase in Expected Current Yield of slightly less than 102bp for TRP.PR.B and slightly more than 102bp for TRP.PR.B.

IV-7

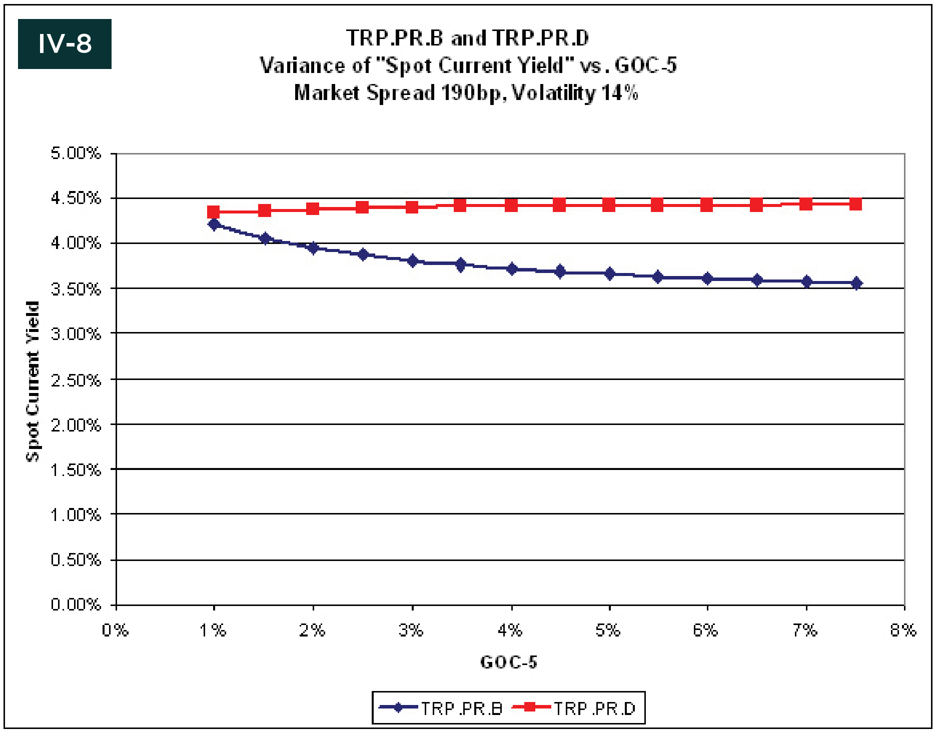


Effect of GOC-5 on "Spot Current Yield"

An objection to the validity of Chart IV-7 is that changes in the GOC-5 rate are not, in fact, realized immediately, but take effect on the next Exchange Date. If the dividend rate on both instruments is held constant while varying the theoretical price in accordance with changes in the GOC-5 rate, we obtain Chart IV-8.

It will be noted that Chart IV-8 embodies an internal contradiction, as the calculation of both Pure Price and Option Value assume that changes in the GOC-5 rate are immediately reflected, whereas the "Spot Current Yield" calculated here assumes that they are not.

In fact, the Theoretical Price should be adjusted to reflect the difference in dividends received between the calculation date and the Reset Date, but I have opted not to do this in the interest of keeping the spreadsheets simple.



The Four TRP Issues in 2013

TRP.PR.D commenced trading on 2013-3-4; using the four issues on dates since that time provides the results shown in Table 1:

Table 1: Results of TRP Implied Volatility Calculations – Best Fit

Date	GOC-5	Market Spread	Volatility	Sum Squared Error
2013-3-28	1.23%	61	27%	0.80
2013-4-30	1.12%	65	28%	0.91
2013-5-31	1.31%	53	32%	1.63
2013-6-28	1.74%	73	32%	1.67
2013-7-31	1.74%	84	32%	1.14
2013-8-30	1.91%	93	32%	0.79
2013-9-12	2.14%	99	32%	2.75

However, it must be noted that not only is the market a rather messy place in terms of pricing, but that local minima exist in the plotting – that is, if one considers a three-dimensional graph, with the x-axis being the Market Spread, the y-axis being the Volatility and the z-axis – the height – being the sum of square errors, there will be more than one point in which any small changes in the x, y, or any combination of the two, will result in an increased height.

I was unable to plot a contour map that showed this to my satisfaction, but the error was calculated for each combination of Market Spread and Volatility in the ranges 50–250bp and 1–40%, respectively. The calculations were performed using the bid prices for the four TRP FixedResets on 2013-9-13, with a GOC-5 rate of 2.12% and a term of three years.

The minimum value for each constant setting with varying values of the other setting was found and results are plotted in Charts IV-12 and IV-13. It is clear that the calculations are susceptible to local minima.

Table 2: Results of TRP Implied Volatility Calculations – Informed Fit

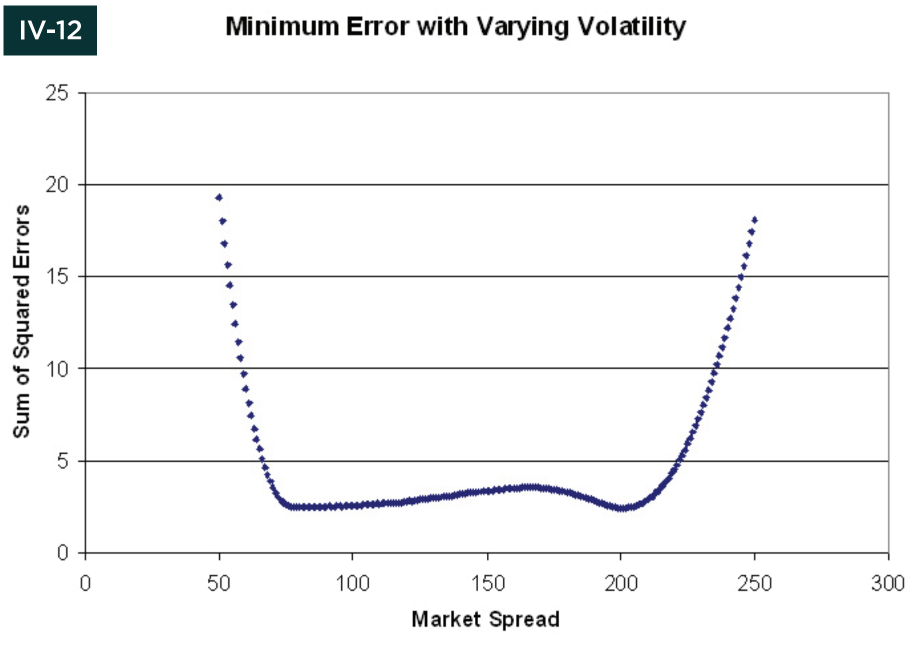
Date	GOC-5	Market Spread	Volatility	Sum Squared Error
2013-3-28	1.23%	79	20%	1.23
2013-4-30	1.12%	86	20%	1.20
2013-5-31	1.31%	89	18%	2.18
2013-6-28	1.74%	127	16%	3.09
2013-7-31	1.74%	159	14%	1.98
2013-8-30	1.91%	175	13%	1.59
2013-9-12	2.14%	182	13%	3.35

I believe that this is particularly true in the current market, which is making something of a transition between pricing based on Current Yield with the expectation of a call on the first reset date and pricing based on the Issue Reset Spread and the expectation that issues will be left outstanding for perpetuity; but this, of course, is mere speculation.

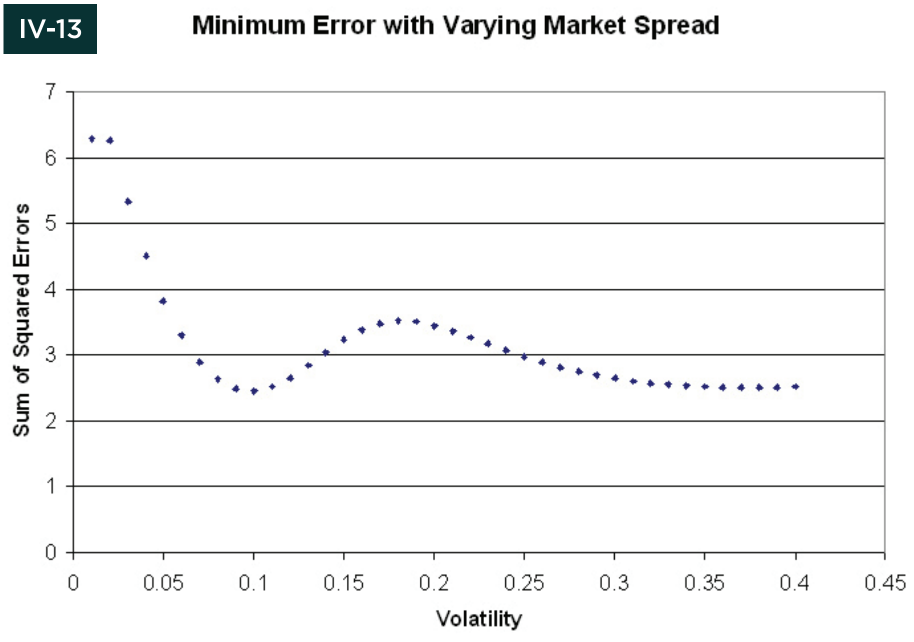
In Table 1, the "best fit" of TRP data on various dates since the issue of TRP.PR.D, the volatility is unreasonably high – particularly considering that it is not the volatility of interest rates in general that is being quantified, merely the volatility of the Market Spread (this will include the term premium, the credit spread and the Seniority Spread, to borrow terminology usually used for PerpetualDiscounts).

More reasonable figures are shown in the results for local minima calculated in Table 2, but it will be observed that the Sum Squared Errors in Table 2 are greater than those of Table 1.

IV-12



IV-13



Investment Conclusions

Calculations of Implied Volatility for FixedResets shows promise as an analytical technique, but the market is not yet sophisticated enough to provide unambiguous data. However, this suggests that investors may achieve excess – albeit irregular – returns by selecting issues that are mispriced according to the theory.