Retirement Withdrawals, Long-term Equity Returns and Annuities

The question of how much money can be drawn annually from an investment portfolio in order to fund retirement is, not surprisingly, a perennial point of interest to investors – and may well become an even greater source of public debate as desperate governments seek to reduce the amount by which support for retirees is socialized.

As has been well-articulated by Dr. Moshe Milevsky,[1] it is not sufficient to project a constant annual return for the portfolio – which is equivalent to calculating Present Value for desired withdrawals – since any plan that involves the reduction of capital will necessarily involve the sale of assets. Given that asset values can fluctuate enormously, it will be clear that a greater proportion of the portfolio will need to be sold when prices are low relative to those times when prices are high. These assets will, of course, not be in the portfolio to enjoy any subsequent rebound of values that may serve to return the portfolio as a whole to its long-term projections – even if those long-term projections turn out to be absolutely accurate.

William Bernstein popularized a retirement calculator that took this effect into account,[2] and retirement calculators with varying features and credibility are now available all over the Internet. Thanks to modern technology, we can now calculate to six decimal places the potential implications of our wild guesses!

The generic fault of these calculators is that most do not account for the cash flow generated by a portfolio that may be considered to be less volatile than the prices of the assets. As discussed at length in the April, 2010, edition of this newsletter, there is a very great difference between an asset that generates no cash, and an asset with the same total return that does generate cash: a 5% withdrawal rate can easily deplete a volatile portfolio with a long-term expected return of 5%; but a 5% perpetual government bond will provide that amount annually[3] in cash, even if the total return for the two portfolios is absolutely identical, year by year.

In short, many calculators treat bonds as "Junior Equities", differing only from their senior cousins by having a lower expected total return and a lower expected dispersion of these returns. I claim that the fact that all bond returns are received annually and in cash[4] (assuming they are bought at par) is a highly significant factor and that this investment attribute is the feature that best distinguishes stocks and bonds.

This essay builds on previous discussion in the April, 2010, and April, 2011, editions of this newsletter, as well as that contained in my essay *Security of Income vs. Security of Principal*.[5] The Hymas Investment Management Inc. retirement calculator (old version available on-line at http://www.prefblog.com/xls/retirementWithdrawals.xls new version at http://www.prefblog.com/xls/retirementWithdrawals_2012.xls) has been refined in various ways and the necessity of these changes is discussed; there is a fairly extensive discussion of "Reversion to the Mean" for equity returns, which is particularly important in the light of OSFI's[6] denial that such a thing exists.

Additionally, I have updated my study of the pricing of annuities to incorporate another year of data collected by Dr. Milevsky's Individual Finance and Insurance Decisions Centre.[7]

Geometric and Arithmetic Means

Some of the figures supplied by the calculator will seem a little odd at first. The raw returns of the Security Template are adjusted by a scaling factor that has the purpose of making the expected return for any twelve monthly samples equal to the specified Expected Annual Return. However, the returns calculated by linking the individual returns geometrically will differ from their arithmetic total:

Geometric Return:
$(1 + r_1) * (1 + r_2) * (1 + r_2) * \ldots * (1 + r_N) - 1$

Arithmetic Average: $(r_1 + r_2 + r_2 + \ldots + r_N)/N$

As discussed in the April, 2011, edition of this newsletter, the two are related by:[8]

Geometric Return = Arithmetic Average – (1/2) Volatility^2

Table A-1 shows the result of ten trials of the "Characterize Data" simulation, in which 8,191 one-year returns have been created in each of ten runs. The geometric average, arithmetic average and standard deviation of these samples are shown for each run.

Table A1: Simulated 1-Year Returns
Bell Curve, 5% Monthly Standard Deviation, 7% Expected Annual Return

Trial	Geometric Mean	Arithmetic Mean	Standard Deviation	Predicted Geometric Mean
1	7.10%	8.50%	17.59%	6.96%
2	6.83%	8.23%	17.56%	6.69%
3	7.60%	9.01%	17.66%	7.45%
4	6.73%	8.09%	17.24%	6.61%
5	7.04%	8.41%	17.34%	6.90%
6	6.96%	8.38%	17.69%	6.81%
7	6.94%	8.34%	17.52%	6.81%
8	6.98%	8.33%	17.22%	6.85%
9	7.33%	8.77%	17.84%	7.18%
10	7.49%	8.90%	17.61%	7.35%

Note that changing the Decile Sampling Factor will have no effect on these results, since the sampling algorithm only becomes operable five years into the simulation.

The "Predicted Geometric Mean is derived by subtracting one-half of the squared Standard Deviation from the Arithmetic Mean.

1 Moshe Milevsky, *Feast or Famine First?*, ResearchMag.com, 2007, available on-line at http://www.ifid.ca/pdf_newsletters/PFA_2007DEC_FEAST.pdf (accessed 2012-4-13)

2 William J. Bernstein, *The Retirement Calculator from Hell*, 1998, available on-line at http://www.efficientfrontier.com/ef/998/hell.htm (accessed 2012-4-13)

3 Unless it's Greek. Or maybe Spanish. Or …

4 Barring default, of course.

5 Available on-line at http://www.himivest.com/media/SecurityIncome.pdf

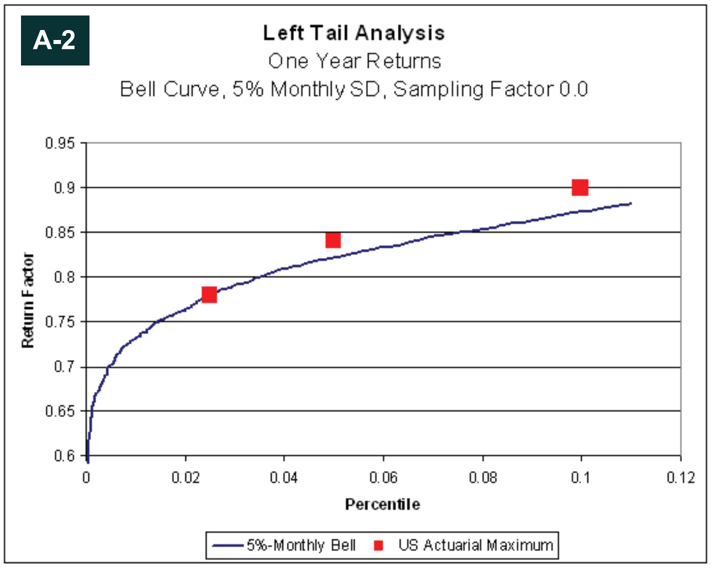
6 Office of the Superintendent of Financial Institutions, which regulates Canada's banks and insurance companies, despite the handicap of having very little expertise.

7 http://www.ifid.ca (accessed 2012-4-13)

8 Peter Ponzo, *Returns and Volatility*, available on-line at http://www.financialwebring.org/gummy-stuff/returns-volatility.htm (accessed 2011-3-31)

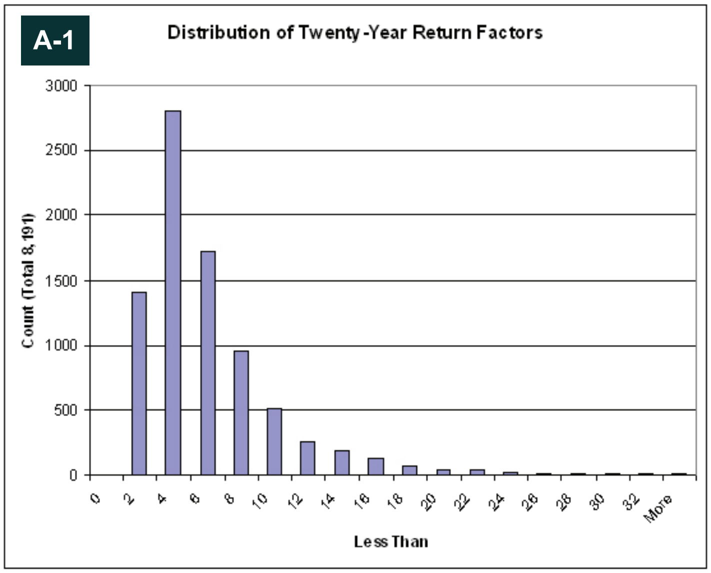
However, as insurance companies and their shareholders discovered during the Credit Crunch, for risk control purposes it is not averages, not even long term averages that are important: what is important is the "left tail" of distributions – that is to say, the lowest 10% of the return distribution.

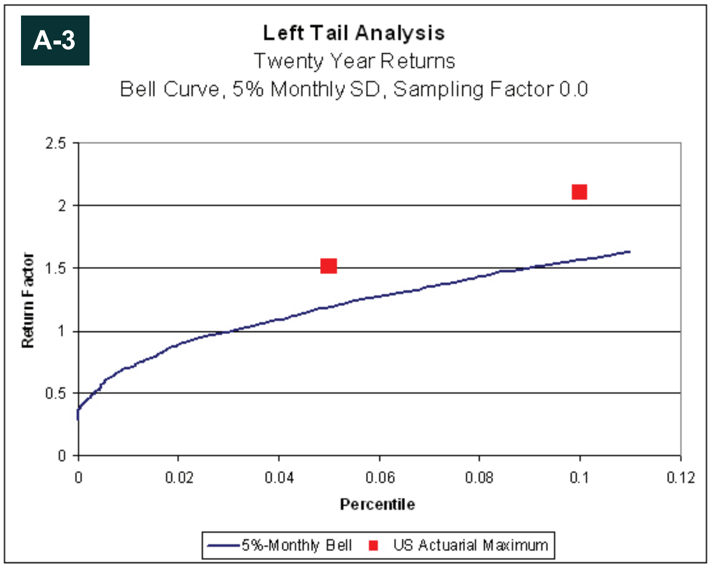
Chart A-2 shows the cumulative distribution chart for one of the simulation runs presented in Table A-1; as may be seen, the relatively simple sampling method gives a good fit to the actuarial requirements for modelled one-year returns recommended by the American Academy of Actuaries in 2005.[9]



Distribution of Twenty Year Returns

The simulations that gave rise to the data in Table A-1 did not stop after one year – they were continued for twenty years of simulated time, with each monthly return being selected randomly from a set of 96 possibilities. The distribution of total return factors for a sample run is shown in Chart A-1, while detail for the left tail is shown in Chart A-3.





As may be seen, the twenty-year returns for the simulation are considerably worse for the standard model (5% monthly standard deviation, 7% annual expectation) than is stated under the actuarial requirements. We cannot simply adjust for this by reducing the monthly standard deviation – that will make the one year returns too stable and, as we have seen, those results are very close to the maxima set by the American Academy of Actuaries already (the Canadian standards require returns that are even worse.

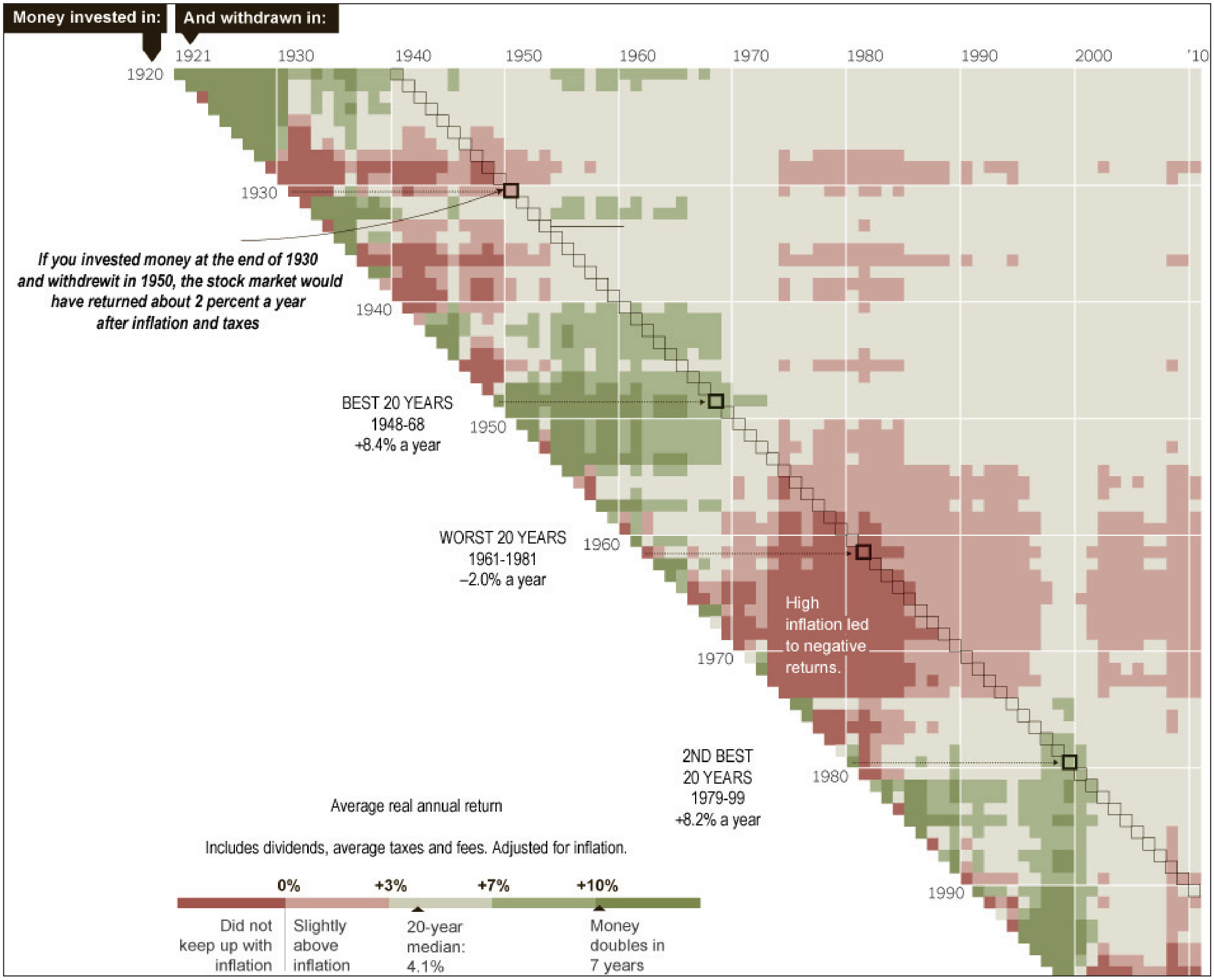
Prior to discussing how this conundrum may be resolved, we will review some theories of long-term equities and their historical basis.

[9] American Academy of Actuaries, *Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products*, June 2005, available on-line at http://www.naic.org/documents/committees_e_capad_lrbc_2_LCASDocFinal.pdf (accessed 2012-3-24)

Long Term Equity Returns

Chart A-4 shows a graphic adapted from a presentation prepared by the New York Times[10] which was in turn prepared from Data provided by Ed Easterling of Crestmont Research.[11] As may be seen, the worst twenty years in what might be called the modern era of stock markets – by which I mean an era which includes a significant middle class which considers investment in public equities to be among its options – occurred in the period 1961–1981, which produced a total return (after inflation and taxes) of -2% – or, to put it another way, resulted in a return factor of 0.98^20 = 0.67, or, if we assume inflation going forward to be 2% annually, 0.96^20 = 0.44, roughly equivalent to the 14th-worst of the 8191 simulations run in the analysis.

A-4



10 The New York Times, *In Investing, It's When You Start And When You Finish*, available on-line at http://www.nytimes.com/interactive/2011/01/02/business/20110102-metrics-graphic.html (accessed 2012-3-25)

11 Ed Easterling, *Unexpected Returns: Understanding Secular Stock Market Cycles*, advertised at http://www.crestmontresearch.com/books/ sold by Amazon at http://www.amazon.com/exec/obidos/ASIN/1879384620/ref=nosim/wwwcrestmontr-20 ISBN-13: 978-1879384620

There are, of course, other ways of looking at long-term equity return projections. Pu Shen of the Kansas City Fed wrote a paper[12] with many informative graphs and notes: *For government bonds, two of the most fundamental long-term risks are changes in long-term inflation trends and the government budget…. For stocks, in contrast, the most fundamental long-term risk is likely to be uncertainty about the trend growth rate of productivity for the overall economy. In the long term, one of the most important fundamental determinants of stock prices is corporate earnings. Corporate earnings are returns to capital. Therefore, earnings should grow at the same rate as the productivity of capital, which in the long run tends to track economywide productivity*.

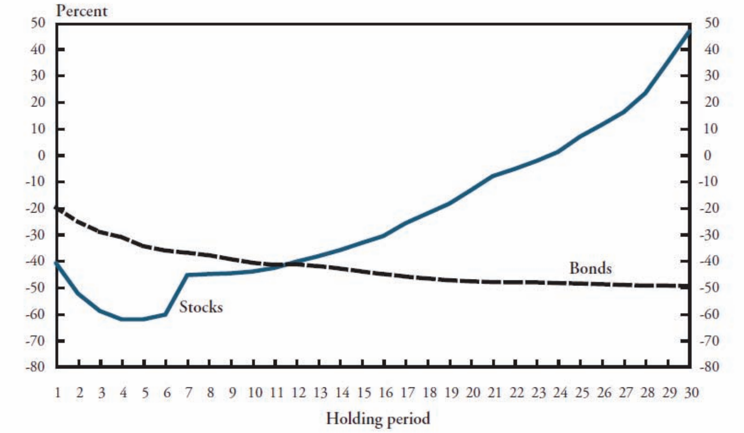
Just as importantly for our purposes, he states: *From 1926 to 2002, the standard deviation of the total return of the stock market index was 21.8 percent for holding periods of one year—but just 5.3 percent for holding periods of ten years. In contrast, for a portfolio of long-term government bonds, the standard deviation of the total return was 9.0 percent for one-year holding periods and 4.0 percent for ten-year holding periods*.

As my final quotation from Dr. Shen's excellent paper, I will highlight: *For investors with holding periods as long as 20 years, stocks were a bit less risky. After 20 years of repeated investment, stocks beat inflation about 98 percent of the time. Over the 2 percent of the time when the real return on stock investments was negative, the worst loss was a cumulative 13 percent. Bond investors, in contrast, were able to beat inflation only 40 percent of the time. Over the other 60 percent of the time, their real returns were negative, and the worst loss was 48 percent (Charts 2 and 3). For holding periods of 20 years, stocks historically outperformed bonds 99 percent of the time (Chart 4). Over the other 1 percent of the time, stock portfolios underperformed bonds, sometimes by as much as 15 percent (Chart 5).*

A-5

WORST REAL RETURN FOR STOCKS AND BONDS

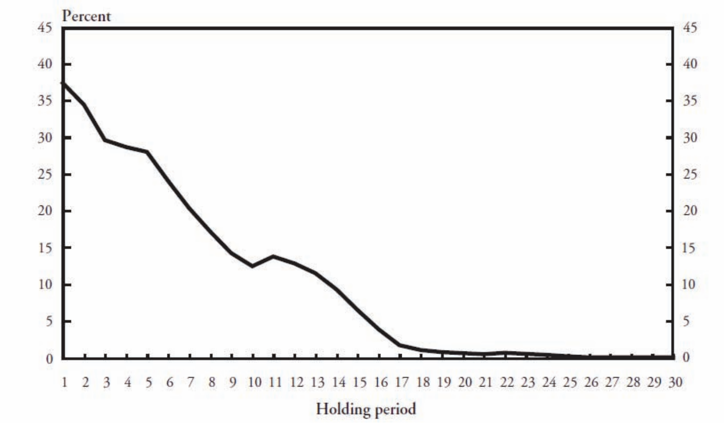
Repeated-investment strategy, by holding period



A-6

FREQUENCY OF STOCKS UNDERPERFORMING BONDS

Repeated-investment strategy, by holding period



Actuarial Methods of Modelling Long Term Equity Returns

The Canadian Institute of Actuaries' 2002 report on segregated fund investment guarantees[13] contains a good amount of discussion on the requirements of any model. They point out, for instance the characteristics of a good pseudo-random number generator (computer algorithms do not generate numbers that are truly random). State dependent models are acceptable; for instance, *a mean-reverting process is state dependent because the future scenarios depend on how the current market variables relate to long-term historical values*.

As is usually the case, however, it was the Americans who produced a report[14] that was of most interest to practitioners. In the course of preparing their recommendations, they note that they used a Stochastic Log Volatility model, chosen because it reflects "volatility clustering" – the observation that extended periods of low volatility (e.g., the Great Moderation) will be followed by relatively brief periods of high volatility (e.g., the Credit Crunch).

[12] Pu Shen, *How Long is a Long-Term Investment?*, Economic Review, First Quarter 2005, Federal Reserve Bank of Kansas City, available on-line at http://www.kansascityfed.org/Publicat/ECONREV/PDF/1q05Shen.pdf

[13] Canadian Institute of Actuaries Tax Force on Segregated Fund Investment Guarantees, *Report*, Canadian Institute of Actuaries, 2002, available on-line at http://www.actuaries.ca/members/publications/2002/202012e.pdf (accessed 2012-4-5)

[14] American Academy of Actuaries, *Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products*, June, 2005, available on-line at http://www.naic.org/documents/committees_e_capad_lrbc_2_LCASDocFinal.pdf (accessed 2012-4-5)

Models of this type were described by Mary Hardy in a referenced paper,[15] and have the following crucial property: *Regime switching allows the stock price process to switch between K regimes randomly; each regime is characterized by different model parameters, and the process describing which regime the price process is in at any time is assumed to be Markov (that is, the probability of changing regime depends only on the current regime, not on the history of the process).*

While I was sorely tempted to develop a model based on this consideration for my retirement withdrawals calculator, I decided instead to implement a simpler approach on the following grounds:

- I feel quite certain that it is possible to devote one's life to the question of developing investment return scenario generators and I must draw the line somewhere
- As noted, each of the K regimes must be separately parameterized; this implies a risk that the model might become over-parameterized in order to produce desired results.
- Further, each of the K parameterizations will need to be justified to users and it is desirable to use a simple model even at the expense of academic rigour in order that the model can be understood intuitively by non-specialist users.

Sampling Error and Reversion to Mean (the "Regression Fallacy")

It is important to remember that it can be very difficult to differentiate between reversion to the mean and sampling error. In 1974, Amos Tversky and Daniel Kahneman[16] related[17] several anecdotes regarding errors that can arise from placing too much confidence in small sample sizes:

Moreover, subjects failed to appreciate the role of sample size even when it was emphasized in the formulation of the problem. Consider the following question:

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital about 15 babies are born each day. As you know, about 50 percent of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50 percent, sometimes lower.

For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?

- *The larger hospital (21)*
- *The smaller hospital (21)*
- *About the same (that is, within 5 percent of each other) (53)*

The values in parentheses are the number of undergraduate students who chose each answer.

Most subjects judged the probability of more than 60 percent boys to be the same in the small and in the large hospital, presumably because these events are described by the same statistic and are therefore equally representative of the general population. In contrast, sampling theory entails that the expected number of days on which more than 60 percent of the babies are boys is much greater in the small hospital than in the large one, because a large sample is less likely to stray from 50 percent. This fundamental notion of statistics is evidently not part of people's repertoire of intuitions.

…

Suppose a large group of children has been examined on two equivalent versions of an aptitude test If one selects ten children from among those who did best on one of the two versions, he will usually find their performance on the second version to be somewhat disappointing. Conversely, if one selects ten children from among those who did worst on one version, they will be found, on average, to do somewhat better on the other version … These observations illustrate a general phenomenon known as regression toward the mean, which was first documented by Galton more than 100 years ago.

P.B. Stark notes[18] that *Failing to account for the regression effect, concluding that something must cause the difference in scores, is called the regression fallacy, The regression fallacy sometimes leads to amusing mental gymnastics and speculation, but can also be pernicious*.

[15] Mary Rosalyn Hardy, *A Regime-Switching Model of Long-Term Stock Returns*, North American Actuarial Journal Society of Actuaries, 2011:5:2, available on-line via http://www.casact.org/dare/index.cfm?abstrID=5496&fuseaction=view (accessed 2012-4-5)

[16] Awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel, 2002 (not precisely a Nobel prize, but functionally equivalent). See http://www.nobelprize.org/nobel_prizes/economics/laureates/2002/kahneman-lecture.html (accessed 2012-4-6)

[17] Amos Tversky & Daniel Kahneman, *Judgment under Uncertainty: Heuristics and Biases*, Science, New Series, Vol 185, No. 4157, 1974-9-27, available on-line at http://psiexp.ss.uci.edu/research/teaching/Tversky_Kahneman_1974.pdf (accessed 2012-4-6)

[18] P.B. Stark, *Chapter 11: Errors in Regression*, available on-line at http://www.stat.berkeley.edu/~stark/SticiGui/Text/regressionErrors.htm (accessed 2012-4-6)

Public Policy Implications of the Regression Fallacy

It is easy[19] to list examples of situations in which the benefit of various initiatives is likely to be overstated, if not completely spurious:

- Improvement in second-year scores on standardized education tests for schools among the worst performing in the first year[20]
- Reduction in traffic accidents in high-risk areas following the installation of speed cameras[21]
- Reduction in wait times for hospital procedures identified as a priority[22]

However, one must be very careful not to overuse the concept of the Regression Fallacy. It is entirely possible that improved (or at least different!) educational techniques can cause improvements in the learning of students; it is entirely possible that the installation of speed cameras changes the behaviour of motorists leading to a reduction in accidents; it is entirely possible that new procedures and resources have improved wait times in a meaningful manner.

In order to differentiate between the Regression Fallacy (in which changes in results are random) and actual reversion to mean (in which there is an actual mechanism that drives results to a certain level) one must formulate various hypotheses and test them in a rigourous manner.

This is particularly the case when it is known that different populations will exist within a sample. For example, the score on College Board's Scholastic Aptitude Tests in the US are estimated[23] to vary *in a range of 30 to 40 points above or below your true ability* (usually!). Each individual student taking multiple tests, will be subject to the regression effect, but the regression will be to the mean for that individual, not to the mean of the total population.

Investment Implications of the Regression Fallacy

The regression fallacy and reversion to mean have many implications for investors.

Manager Selection: John Bogle has made a career of decrying active management and has strong views on manager selection:[24] *Reversion to the mean – the law of gravity in the financial markets that causes funds that are up to go down, and funds that are down to go up – is clear, quantifiable, and apparently almost inevitable.* It should be noted that in this quote, Mr. Bogle has used "reversion to the mean" where this essay would express the concept as "regression to the mean". "Mean Reversion" is used in this essay to imply that there is some mechanism that forces the long-term value to the mean.

The regression fallacy can also work the other way: I have been told a story about an institutional portfolio manager who had just completed his third successive year of poor relative returns when he got a call from his largest client to attend a meeting of the client's investment committee. Certain that he was about to be fired, he attended – and was rewarded with an increased allocation of cash on the grounds that his investment style had been unfashionable for so long, it was bound to do better in the future!

In fact, of course, if relative investment returns are completely random in Mr. Bogle's homogeneous universe, then it doesn't matter which manager you choose, first quartile or fourth – the probability distributions going forward will be identical.

Contrarian Investment Strategies: I often see investment managers describing themselves as contrarian – it's a good marketing gimmick, redolent of rugged individualism and fierce independence. However, if they say nothing more than "contrarian" and do not provide sufficient details about their valuation models to provide comfort, I lose interest very quickly. An uninformed contrarian strategy is simply an example of the regression fallacy.

OSFI's Prohibition of Mean-Reverting Models

OSFI has leapt into the debate, stating[25] that it *has decided that it will not review requests for approval to use internal models that incorporate mean reversion in equity returns*. To support this view, it has prepared an internal paper[26] with all the rigour that has become such a hallmark of Canadian financial regulation.[27]

OSFI claims that *the existence of mean reversion would contradict the Efficient Markets Hypothesis, which states that stock price movements cannot be predicted based on previous movements. If they could be, then traders would be able to earn excess returns (i.e. in excess of the overall market return) by buying stocks that have had lower-than-average returns in prior periods and short selling stocks that have had higher-than-average returns in prior periods. However, if there really is money to be made by following this strategy, what will happen (according to the hypothesis) is that traders alert to this opportunity will, in the current period, push up the prices of stocks that underperformed in prior periods and pull down the prices of stocks that overperformed in prior periods, so that the opportunity to make an excess profit will vanish. For this reason, tests of whether mean reversion exists are considered to be a subfield of tests of market efficiency*.

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[19] Even Wikipedia can do it.

[20] Wikipedia, *Regression Towards the Mean: Regression Fallacies*, available on-line at http://en.wikipedia.org/wiki/Regression_toward_the_mean#Regression_fallacies (accessed 2012-4-6)

[21] Wikipedia, *Regression Towards the Mean: Regression Fallacies*, available on-line at http://en.wikipedia.org/wiki/Regression_toward_the_mean#Regression_fallacies (accessed 2012-4-6)

[22] Canadian Institute for Health Information, *Most patients receiving care within recommended wait times for priority areas*, Press Release, 2010-3-24, available on-line at http://www.cihi.ca/CIHI-ext-portal/internet/en/Document/health+system+performance/access+and+wait+times/RELEASE_24MAR10 (accessed 2012-4-6)

[23] College Board, *Understanding Your Scores*, available on-line at http://sat.collegeboard.org/scores/understanding-sat-scores (accessed 2012-4-6)

[24] John C. Bogle, *Common Sense on Mutual Funds: New Imperatives for the Intelligent Investor*, John Wiley & Sons, 2000, ISBN-13: 978-0471392286 excerpted at http://finance.yahoo.com/funds/how_to_choose/article/100564/Do_Not_Overrate_Past_Fund_Performance (accessed 2012-4-7)

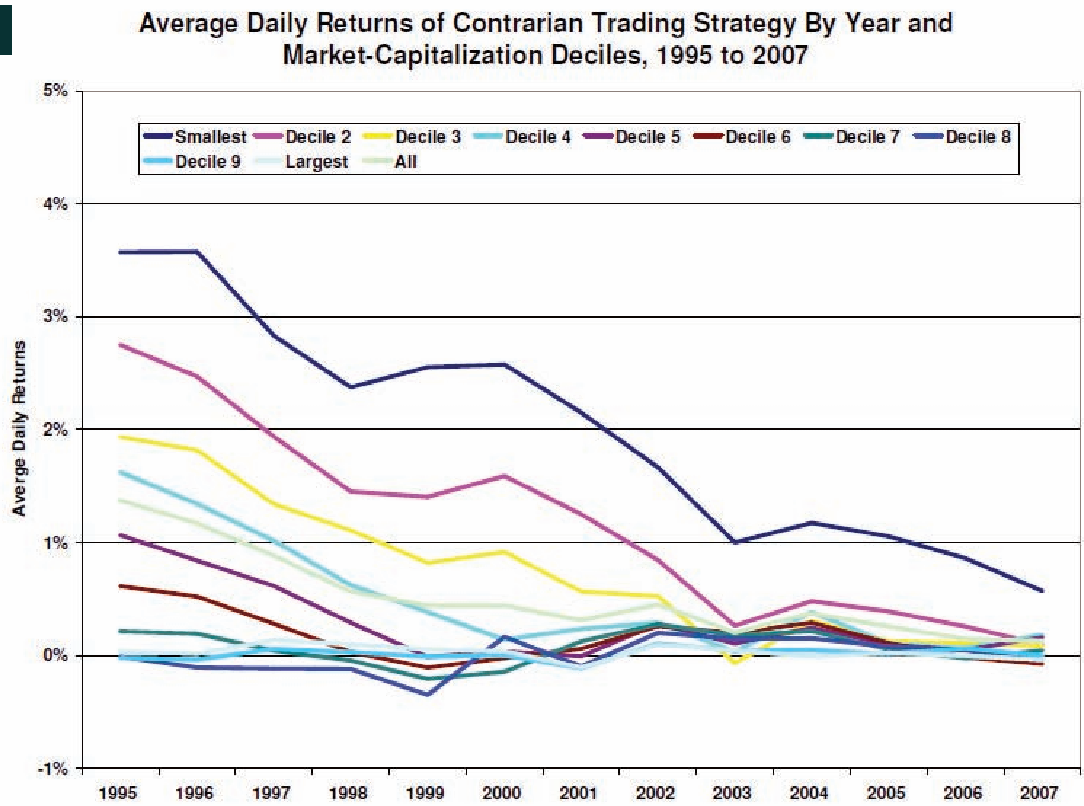
[25] Mark Zelmer, *Mean Reversion in Models of Equity Returns*, memorandum, Office of the Superintendent of Financial Institutions, March, 2012, available on-line at http://www.osfi-bsif.gc.ca/app/DocRepository/1/eng/notices/osfi/mnrv_let_e.pdf (accessed 2012-4-6)

[26] Daniel Mayost, *Evidence for Mean Reversion in Equity Prices*, Office of the Superintendent of Financial Institutions, March, 2012, available on-line at http://www.osfi-bsif.gc.ca/app/DocRepository/1/eng/notices/osfi/mnrv_e.pdf (accessed 2012-4-6)

[27] That is to say, none.

This is a rather surprising paragraph for several reasons: firstly, it accepts the Efficient Markets Hypothesis (EMH) as gospel, while the concept has been under attack for quite some time.[28] In addition, this contrarian strategy has been a favourite of hedge funds for many years[29]; while there has been a clear diminution of returns over the sample period culminating August 2007,[30] it is clear that a naïve contrarian strategy of buying issues that declined on the previous day and shorting issues that advanced was clearly profitable over a much larger timescale than envisaged by the EMH; the profits, and their decline over time, are shown in Chart A-9.

A-9



Another problem with the statement is that it does not address the issue. The author dismisses the concept of equity return reversion as it is applied to earning excess alpha – that is, the potential for outperforming the market consistently based on taking different investment views on sub-populations of the broad market. However, while I am sure that insurance companies have no aversion to earning risk-free excess returns, the modelling at issue is whether market returns as a whole will exhibit mean-reversion, which is a different question.

OSFI's paper attempts to buttress its position by citing option prices: *If market participants truly believe that equity markets revert to the mean over the long run then this should be observable in option prices, which reflect the market price of hedging long-dated equity guarantees. In particular, the cost of purchasing put option protection against a long-run decline in equity markets should be minimal. However, the cost of purchasing such protection is in fact very expensive, if it is available at all, which suggests that the market does not believe in mean reversion*.

This rather broad statement contradicts without justification the finding by Bayliffe and Pauling[31] that: *Recent over-the counter (OTC) quotes for long-term equity puts have implied volatilities that increase with term in contrast to volatilities past behaviour. This has several contributing causes, not least being that market participants do not use a Black Scholes model to price these options. Models actually used are likely to reflect the non-normality of market returns, the high transactions costs of adjusting hedges over long time periods and a profit margin*.

28 Burton G. Malkiel, *The Efficient Market Hypothesis and Its Critics*, CEPS Working Paper No. 91, April 2003, available on-line at http://www.vixek.com/Efficient%20Market%20Hypothesis%20and%20its%20Critics%20-%20Malkiel.pdf (accessed 2012-4-6). This paper defends the EMH. but references and discusses the attacks.

29 As discussed in the December, 2009, edition of this newsletter

30 Amir E. Khandani and Andrew W. Lo, *What Happened To The Quants in August 2007?*, available on-line at http://web.mit.edu/alo/www/Papers/august07.pdf (accessed 2009-12-9)

31 David Bayliffe and Bill Pauling, *Long Term Equity Returns*, Submitted to the 2003 Stochastic Modeling Symposium Call for Papers Program, available on-line at http://www.towersperrin.com/tp/getwebcachedoc%3Fwebc=TILL/USA/2003/200309/Long_Term_Equity_Returns.pdf (accessed 2012-4-6)

The issue is particularly interesting in light of Berkshire Hathaway's well publicized writing of put options on major market indices, which has been discussed extensively,[32] albeit not by OSFI.

The OSFI paper concludes with an examination of the Draw-Down model proposed[33] in 2002, in which the *model incorporates a mean reversion assumption is by adding a term to the expected return equation that is proportional to the difference, if positive, between the value of the index at its last peak and its current market value*. The examination is so pointed that I cannot help but think that the policy as a whole is a response to a specific insurer using a model based on this specific technique; but the counter-arguments presented are lamentable:

- The Dow Jones Industrial Average did not return to its 1929 high until 1954, representing a recovery time of 25 years.
- The NASDAQ Composite Index, after 12 years, is still only at 60% of its January 2000 high.
- The Nikkei 225 index, 22 years after its peak at the end of 1989, has still not recovered and is currently worth less than a quarter of its all-time high.

Disproof by counterexample is simply not an appropriate technique in this case; if I should claim that the next winning roulette number will probably not be 23-Red, then no number of anecdotes citing past counterexamples will suffice to disprove my claim – a more detailed statistical examination will be required. Mary Hardy et al.'s uncited examination of the model[34] provides a far more rigourous critique: *the effect on long-term returns is dramatic as the additional term causes a significantly thinner left tail for longer accumulation factors than the RSLN model. There are open questions here about the effect of survivorship – that one way for the recovery of an index such as the S&P 500 is the replacement of failing stocks with healthier alternatives, giving the index a better return than would be achieved by an investor*.

It will further be noted that mean-reverting models were explicitly allowed by the task force that originated OSFI's calibration requirements: *State dependent models relate the change from one period to the next to current market levels or recent market performance. For example, a mean-reverting process is state dependent because the future scenarios depend on how the current market variables relate to long-term historical values. State dependent models are not required, but are acceptable if they are justifiable based on the historical data and meet the calibration criteria*. It will also be noted that the equivalent American report states *State dependent models are not prohibited, but must be justified by the historic data and meet the calibration criteria. To the degree that the model uses mean-reversion or path dependent dynamics, this must be well supported by research and clearly documented in the Memorandum supporting the required actuarial certification*. To overturn the professional judgment of the task force with no more justification than OSFI's supporting memorandum is reckless public policy and an insult to the original task force members.

In summary, OSFI's prohibition of mean-reverting models must be regarded as without value as an indicator of real-world phenomena. While I am pleased that OSFI is attempting to justify its pronouncements with an effort at examination of actual data, the schoolboy essay used in this instance is a disgrace. It will be remembered that the consensus explanation for the resilience of Canadian banks during the credit crunch had less to do with OSFI's wisdom than with the very stable funding granted by complacent Canadian depositors.[35]

My Own Views on Mean Reversion

For long term returns, I agree with Shen, who states: *For stocks, in contrast, the most fundamental long-term risk is likely to be uncertainty about the trend growth rate of productivity for the overall economy. In the long term, one of the most important fundamental determinants of stock prices is corporate earnings. Corporate earnings are returns to capital. Therefore, earnings should grow at the same rate as the productivity of capital, which in the long run tends to track economywide productivity*.

More formally, I would state this as:

$$\text{GDP} \times \frac{\text{Corporate Profits}}{\text{GDP}} \times \frac{\text{Public Corporation Profits}}{\text{Corporate Profits}} \times \frac{\text{Established Public Corporation Profits}}{\text{Public Corporation Profits}} = \text{Established Public Corporation Profits}$$

The first term is the simple GDP, which – in the absence of an apocalypse – will be a highly bounded value. I argue that there is a natural growth rate to GDP, based on the fact that there are more people in the world today thinking about how to do things better than there have been at any time in human history. A civilization of subsistence farmers – which was essentially the state of the world until the Industrial Revolution – cannot afford a class of thinkers of a significant size. We can.

The second term is the proportion of GDP represented by corporate profits. Chart A-10, taken from the New York Times,[36] shows the evolution of this fraction in the US for the past fifty years.

32 E.g, Thomas Courtright, *Incite: Berkshire Hathaway's option valuation lesson*, CFOZone, 2010-1-27, available on-line at http://www.cfozone.com/index.php/Incite/Incite-Berkshire-Hathaway-s-option-valuation-lesson.html (accessed 2012-4-6)

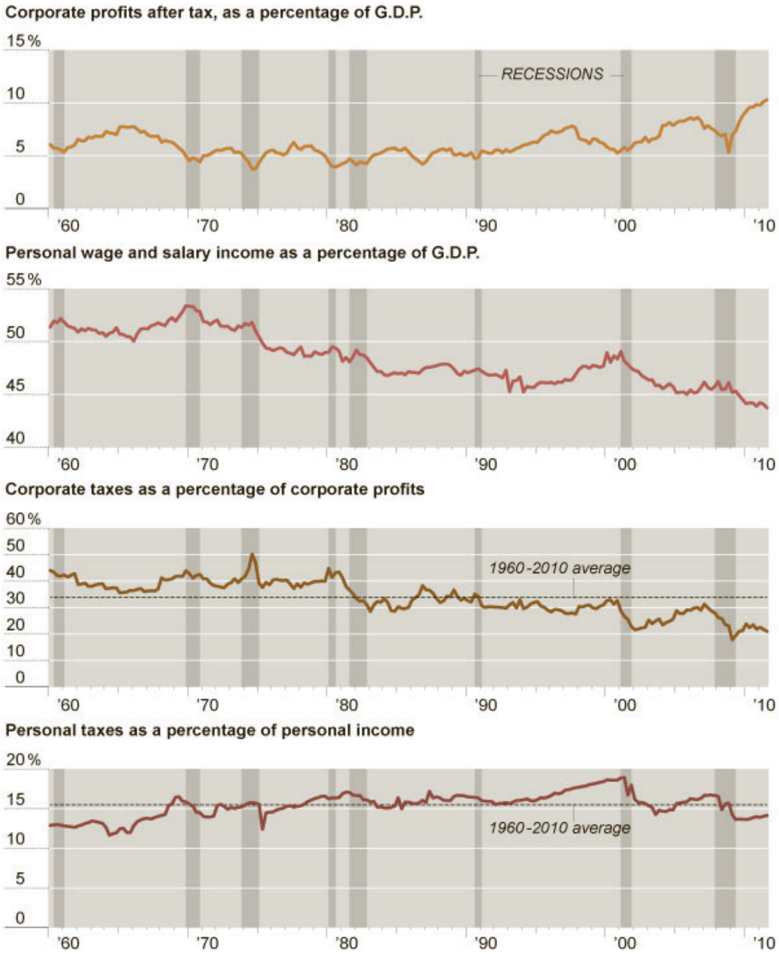
33 Panneton, 2002, *Mean-Reversion in Equity Models in the Context of Actuarial Provisions for Segregated Fund Investment Guarantees*, Segregated Funds Symposium Proceedings, Canadian Institute of Actuaries, available on-line via http://www.actuarialfoundation.org/research_edu/scolastic03.htm (accessed 2012-4-6)

34 Mary R. Hardy, R. Keith Freeland and Matthew C. Till, *Validation of Long-Term Equity Return Models for Equity-Linked Guarantees*, North American Actuarial Journal, Volume 10, Number 4, available on-line at http://www.actuaries.ca/afc/documentations/Mary_Hardy_naaj0604_3.pdf (accessed 2012-4-6)

35 Lev Ratnovski, Rocco Huang, *Why Are Canadian Banks More Resilient?*, International Monetary Fund Working Paper No. 09/152, 2009-7-1, available on-line via http://www.imf.org/external/pubs/cat/longres.cfm?sk=23040.0 (accessed 2012-4-8)

36 New York Times, *Profits Are High, Wages Are Low and Taxes Are Below Average*, available on-line at http://www.nytimes.com/interactive/2011/11/25/business/profits-are-high-wages-are-low-taxes-are-below-average.html?ref=business

A-10



The third term in the series relates the proportion of public company profits to total profits; clearly, this value must be bounded in the range [0,1] over the long term.

The fourth term allows for slippage – upstart competitors eating the lunch of previously established firms. I suggest that this value will be fairly constant over the long term.

Finally, the stock market valuation of these profits will vary enormously over the short term, but in a world where inflation is bounded by central bank policy (and inflation may become formally mean reverting itself should Price Level Targetting[37] come into vogue) I suggest that the Price/Earnings multiple will not only be bounded,[38] but will be mean reverting, since extreme values will distort the economy.

I will note that according to Manzan:[39] *market price experiences significant swings away from its fundamental valuation but reverts back in the long-run*, which is in accord with my own less academic observations. In addition, the results of Nielsen and Olesen[40] are of great interest: *Hence, we conclude that the weak evidence of mean reversion presented in table 5 is (mainly) a result of serial correlation in the high return-high volatility state which has dominated the most recent decades*.

Faugère and Erlach have presented[41] a theory of stock market valuation that values the S&P 500 *as a function of two components, (1) a perpetuity based on after-tax forward earnings, and (2) the after-tax present value of growth opportunities… We introduce a novel condition that the present value of expected growth opportunities mean-reverts to zero, which is consistent with the evidence of mean reversion of aggregate earnings growth and corporate profitability, as well as the evidence in favor of mean reversion of returns*.

Thus, I consider mean reversion to be perfectly acceptable when modelling equity returns over the long term, while noting there will be huge uncertainty when predicting returns over any defined period of time, such as the constant 20-year periods highlighted in Chart A-4, or the variety of constant periods examined by Shen illustrated in Charts A-5 and A-6.

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37 For a review of Price Level Targetting, see Bank of Canada, *Review*, Spring 2009, available on-line at http://www.bankofcanada.ca/wp-content/uploads/2010/06/review_spring09.pdf (accessed 2012-4-7)

38 John Y. Campbell, Robert J. Shiller, *Valuation Ratios and the Long-Run Stock Market Outlook*, Journal of Portfolio Management, Winter, 1998, available on-line at http://www.econ.yale.edu/~shiller/online/jpmalt.pdf (accessed 2012-4-7)

39 Sebastiano Manzan, *Nonlinear Mean Reversion in Stock Prices*, Quantitative and Qualitative Analysis in Social Sciences, Vol. 1, Issue 3, 2007, available on-line at http://www.qass.org.uk/2007/vol1_3/paper_1-manzan.pdf (accessed 2012-4-7)

40 Steen Nielsen & Jan Overgaard Olesen, *Regime-Switching Stock Returns and Mean Reversion*, Institut for Nationaløkonomi, Working Paper 11-2000, available on-line at http://openarchive.cbs.dk/bitstream/handle/10398/7529/wpec112000.pdf?sequence=1 (accessed 2012-4-8)

41 Christophe Faugère and Julian Van Erlach, *A Required Yield Theory of Stock Market Valuation and Treasury Yield Determination*, New York University Salomon Center and Wiley Periodicals, Inc., 2009, available on-line at http://requiredyieldtheory.com/images/RYTDec2008FaugereVanErlach.pdf (accessed 2012-3-25)

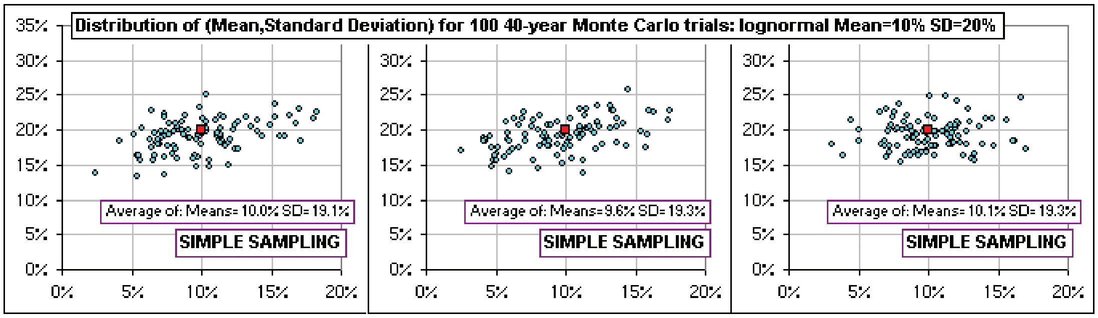
Latin Hypercube Sampling Method

This method of sampling (also called "Stratified Sampling") was discussed in the April, 2011, edition of this newsletter, leaning very heavily on the explanation of Dr. Peter Ponzo.[42]

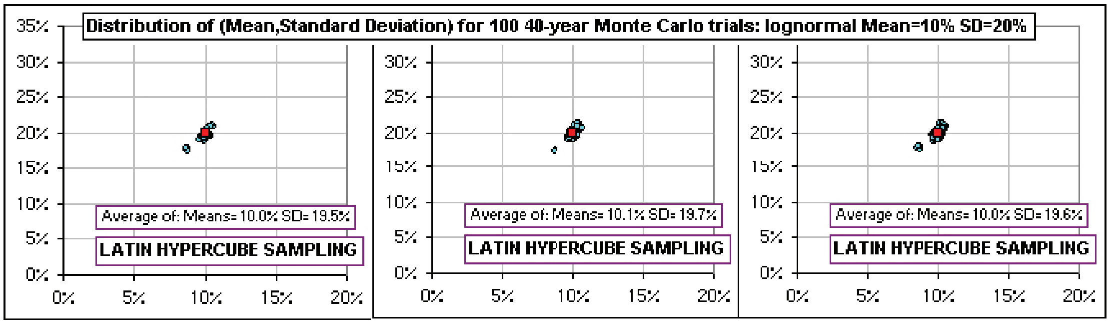
Briefly, Latin Hypercube sampling involves dividing the full spectrum of possible sample values into N regions and ensuring that one sample point is taken from each of these regions. This allows the mean and standard deviation of the entire sample to be calculated accurately using fewer samples than would be required if the sampling method was completely random.

As shown by Dr. Ponzo's examples, shown here as Charts A-11 and A-12, this results in a moderate sample size (40 samples) resulting in much more precise estimates of the mean and standard deviation of the universe than results from the random sampling of this universe.

A-11

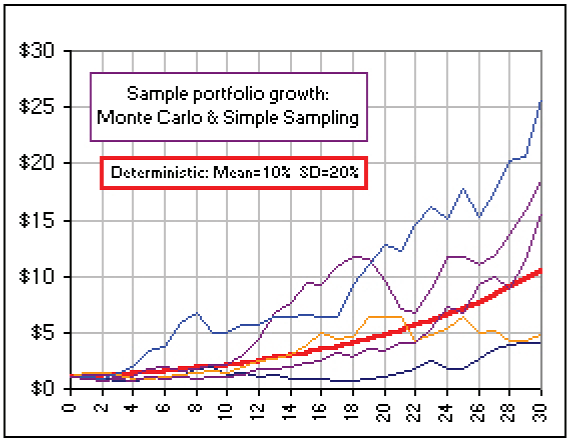


A-12

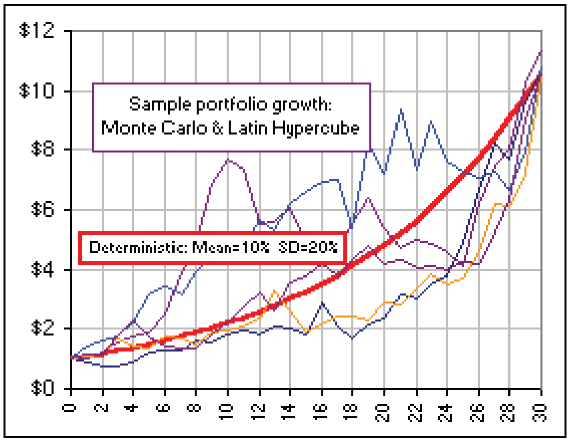


This is all very well and good, but as Dr. Ponzo points out, the method is too deterministic for the intended purpose: Chart A-13 shows the paths of five Monte Carlo simulations generated by simple sample (with the thick red line representing constant growth equal to the desired mean); Chart A-14 is a similar plot with the five simulations run using Latin Hypercube sampling.

A-13



A-14



It should be clear that generating Monte Carlo portfolio simulations using Latin Hypercube sampling yields results that are too deterministic – or, to use the language of this essay, are too strongly mean reverting.

[42] Peter Ponzo, *Latin Hypercube and sampling stuff*, available on-line at http://www.financialwebring.org/gummy-stuff/latin-hypercube.htm (accessed 2012-4-7)

Modified Latin Hypercube Method

The problem with the Latin Hypercube method as described is that it is too sharp-edged – by the time you are computing the last few periodic returns, the regions to be sampled are well-defined.

My objection to this is related to my objection to most stock-screening methods: the edges are too sharp. For instance, if we consider a stock-screener that requires Price-Earnings ratios of less than 15 and Price-Book ratios of more than 1.0, then we will reject a stock that has a P/E of 15.1 and a P/B of 3.0. This does not necessarily make a lot of sense – it is better if we say make the requirement of one dependent upon the other, or define an objective function that combines the two elements in some manner so that deficiencies in one may be compensated for by excessive scores in the other. I've seen objective functions developed for cat breeding[43] that made more sense than any stock screen I've ever examined.[44]

Accordingly, my philosophy of market valuation is that as little as possible should be strictly forbidden. Instead, undesirable characteristics should simply give rise to penalties in the objective function, allowing stellar qualities in other attributes the potential to compensate for the weaknesses. It is with this in mind that I modified the Latin Hypercube methodology described above with the aim of nudging, rather than forcing, the sampling to be more evenly distributed.

In this methodology, no adjustments are applied to the random sampling of possible returns for the first sixty months of each simulation; however, these returns are ranked in deciles and a record kept of the sampling by decile.

Subsequent to the first sixty months, the raw random number is mapped onto an adjusted number line; if a decile has been under-sampled, then more than 1/10 of the possible random numbers will map to that decile; over-sampling will result in fewer than 1/10 of the random numbers mapping to that decile. The relevant code is:

```
(1) For aspCounter = 0 To 9
    aspRawIncrement(aspCounter) = 0.1 + (0.1 – aspDecilePercentage(aspCounter)) * decileSamplingFactor

    If aspRawIncrement(aspCounter) < 0.01 Then
        aspRawIncrement(aspCounter) = 0.01

    End If
    aspRawIncrementTotal = aspRawIncrementTotal + aspRawIncrement(aspCounter)

    Next aspCounter

(2) For aspCounter = 0 To 9
    aspIncrementMap(aspCounter) = aspRawIncrement(aspCounter)/aspRawIncrementTotal

    Next aspCounter

(3) aspCumulativeTotal = 0#
    For aspCounter = 0 To 9
        If aspPoint > aspCumulativeTotal Then
            If aspPoint < aspCumulativeTotal + aspIncrementMap(aspCounter) Then
                aspResult = 0.1 * aspCounter + (aspPoint – aspCumulativeTotal) * 0.1/
    aspIncrementMap(aspCounter)

    End If
    End If
         aspCumulativeTotal = aspCumulativeTotal + aspIncrementMap(aspCounter)

    Next aspCounter
```

Notes

(1) Raw increments are calculated by calculating the difference between the expected proportion of hits (1/10 in each decile) and the actual proportion. This difference is then multiplied by the Decile Sampling Factor and thus creates the new probability that the decile will be selected (with a minimum of 1%).

Thus, if the actual frequency is 14%, then the difference between expected and actual is 4%, and the new probability of that decile being sampled is 6% (assuming that the Decile Sampling Factor equals 1).

(2) Since a minimum has been applied to the calculation of Note 1, the resultant probability set must be renormalized.

[43] Roy Robinson, *Genetics for Cat Breeders*, 2nd edition (1989), Pergamon Press, ISBN 0-08-021209-3

[44] Cat breeders are generally playing with their own money.

(3) The random input between 0 and 1 is then mapped onto the new probability distribution. Say, for instance, that the random input is 0.25, but due to under-sampling of the first two deciles, these two deciles both have adjusted probabilities of 0.15.

The input figure of 0.25 maps into the second decile (which captures all random inputs between 0.15 and 0.30) and is two-thirds of the way through its length. Thus, the random input 0.25 will map onto adjusted output of 0.16.

The regular portfolio returns sampling routine will then produce the return equal to the 16th percentile of the ranked returns.

A Drawback of the Decile Sampling Factor Implementation

The actuarial requirements for relatively short periods (particularly one-year) are meant to be applicable for any given year in a long-term simulation, not merely the first five years.

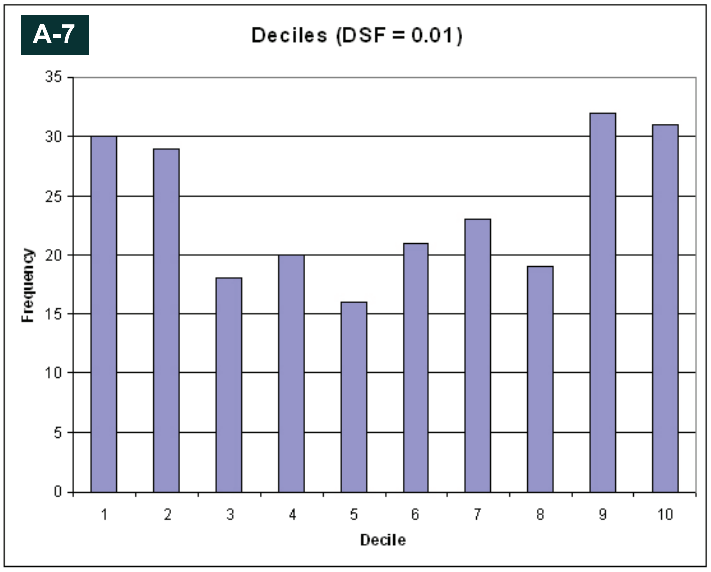
However, it will be clear that the required extreme left tail of any yearly distribution will be populated only by simulations in which the lower deciles are over-sampled relative to the higher deciles; this will be much less likely when the effects of the Decile Sampling Factor are applicable.

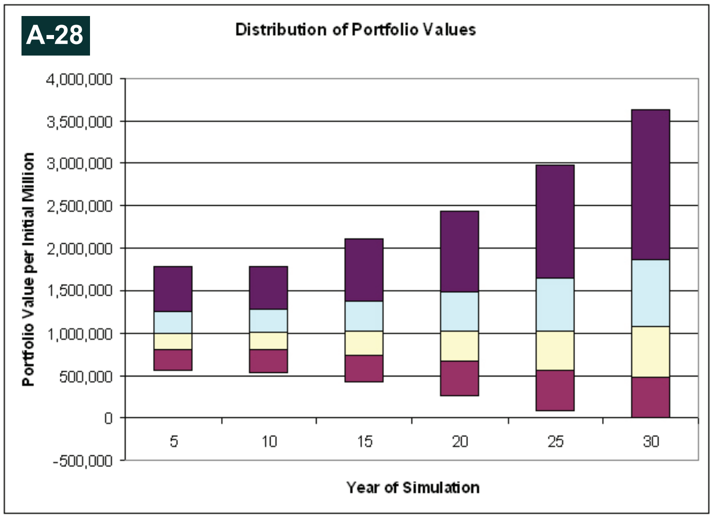
Thus, while the distributions in the first five years of each simulation will meet the specifications, it is unlikely that the distributions in succeeding years will achieve such extreme results – on either the positive or negative side. This is most apparent in years 5-10 of the simulations, in which the prior state will be at its most random, and the relatively small number of monthly samples will increase the effect of the decile sampling algorithm. Chart A-28 shows the evolution of portfolio dispersion from one of the runs used to create Table A-19 (the run defined by a 50/50 mix of stocks and bonds, with an annual withdrawal rate of 5%). It is evident that the dispersion measured at year 10 is not much different from the dispersion at year 5.

I can only console myself with the observation that poor performance in the earlier years of a simulation has a much greater effect on final returns than it does in later years,[45] which mitigates the effect on later results. It will also be noted that if the performance in the early years of a simulation should be better than expected, the effect of the Decile Sampling Factor will be to ensure that this is followed by poor performance later on.

Effect of Changing the Decile Sampling Factor

Chart A-7 shows the sampling frequency per decile for the first 239 months (I didn't bother recording the last month!) for a run with the Decile Sampling Factor set to 0.01.





To quantify the evenness of the sampling, we will define a score S, such that:

S = ∑ (a–p)^2

Where a = the actual number of samples
p = the predicted number of samples

[45] Moshe Milevsky, *Feast or Famine First?*, available on-line at http://www.ifid.ca/pdf_newsletters/PFA_2007DEC_FEAST.pdf (accessed 2012-4-8)

The calculation of S for the simulation shown in Chart A-7 is shown in Table A-2:

Table A-2: Calculation of S for Sample Run with DSF = 0.01

Decile	Actual Samples	Predicted Samples	Squared Difference
1	30	23.9	37.2
2	29	23.9	26.0
3	18	23.9	34.8
4	20	23.9	15.2
5	16	23.9	62.4
6	21	23.9	8.4
7	23	23.9	0.8
8	19	23.9	24.0
9	32	23.9	65.6
10	31	23.9	50.4
Sum	**239**	**239**	**324.9**

Having defined our terms, we can now investigate the effect of changing DSF on the characteristics of a simulation. All simulations were performed for 8,191 runs (of which one was chosen randomly for the calculation of S in Table A-3), with the distribution defined with a monthly Standard Deviation of 5% in a bell-shaped curve, and an expected return of +7% annually.

As may be seen, we are able to achieve good agreement with the American actuarial requirements with a DSF in the range of 0.3–0.4. However, it will also be noted that the New OSFI standard places a lower maximum value on the one-year returns, while allowing a higher return for the ten year period. While I feel that there are sufficient degrees of freedom (monthly standard deviation, expected annual return and Decile Sampling Factor) in the spreadsheet to allow for modelling returns close to these standards, I suggest that it is both more realistic and more intellectually satisfying to discard the notion of a bell-shaped monthly return distribution and use a distribution derived from actual market data.

Table A-3: Calculations of S for Various Decile Sampling Factors (DSF)

DSF	S
0.01	324.9
0.10	254.9
0.20	170.9
0.30	142.9
0.40	88.9
0.50	112.9

Table A-4: Effect of Changing DSF on 5%-ile value of Characterization Bell Curve, 5% Monthly Standard Deviation, 7% Expected Annual Return

Description	1 Year	5 Years	10 Years	20 Years
US Actuarial Maximum	**-16%**	**-19%**	**-6%**	**+51%**
DSF = 0.01	-18%	-22%	-14%	+20%
DSF = 0.10	-17%	-22%	-11%	+30%
DSF = 0.20	-18%	-23%	-8%	+37%
DSF = 0.30	-18%	-23%	-6%	+42%
DSF = 0.40	-18%	-22%	-3%	+55%
DSF = 0.50	-18%	-22%	+1%	+61%

Chart A-8 shows the sampling distribution from one of the runs with DSF = 0.50. Note that this is just one of the 8,191 runs that produce the results displayed in Table A-4.

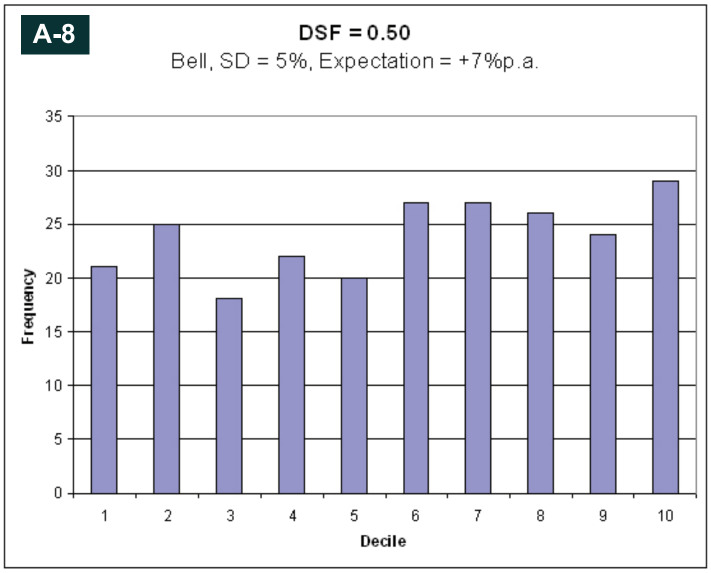
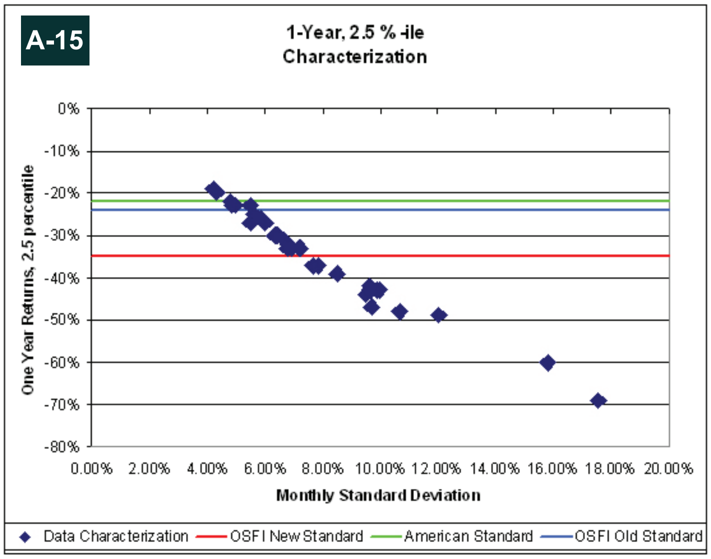
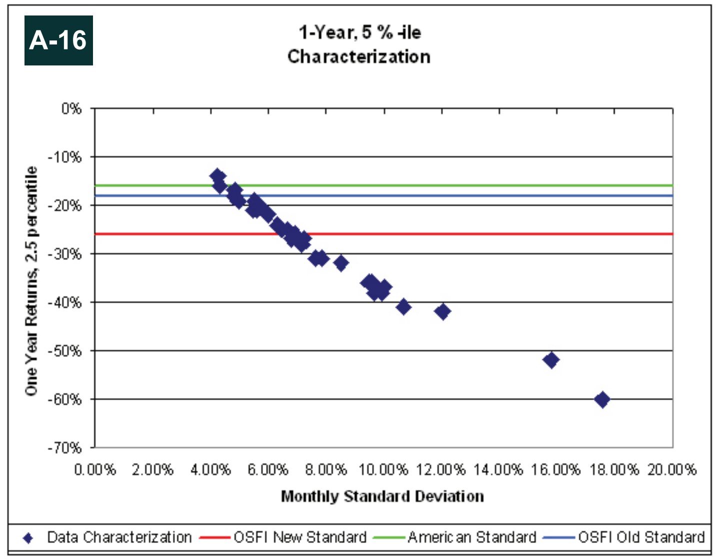


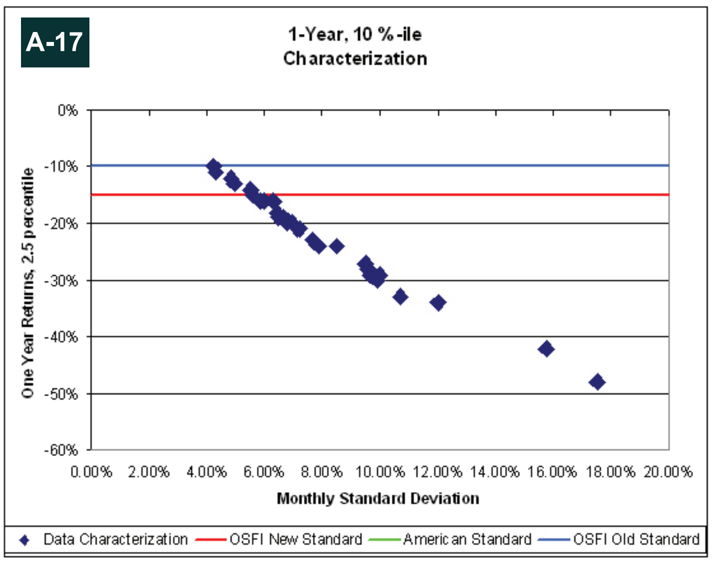
Table A-12: Characterization of Security Templates									
Stock	**StDev**	**Skew**	**Kurtosis**	**1-Year 2.50%**	**1-Year 5%**	**1-Year 10%**	**5-Year 5%**	**10-Year 5%**	**20-Year 5%**
OSFI New				**-35%**	**-26%**	**-15%**	**-15%**	**5%**	**N/A**
American				**-22%**	**-16%**	**-10%**	**-19%**	**-6%**	**51%**
OSFI Old				**-24%**	**-18%**	**-10%**	**-15%**	**5%**	**N/A**
XIU	4.22%	-1.07	3.37	-19%	-14%	-10%	-14%	2%	58%
ENB	4.32%	-0.35	0.25	-20%	-16%	-11%	-17%	-4%	44%
XFN	4.82%	-0.32	2.31	-22%	-17%	-12%	-20%	-11%	28%
XRE	4.84%	-1.4	5.72	-23%	-18%	-12%	-19%	-5%	42%
BNS	4.95%	-0.35	0.89	-23%	-19%	-13%	-22%	-12%	27%
BCE	5.49%	-1.54	8.34	-27%	-21%	-14%	-23%	-12%	37%
CVE	5.50%	0.3	-0.28	-23%	-19%	-14%	-26%	-24%	-4.00%
RY	5.60%	-0.01	1.83	-25%	-21%	-15%	-29%	-22%	6%
CNR	5.85%	0.05	-0.11	-26%	-21%	-16%	-31%	-27%	1%
TD	6.00%	-0.05	1.06	-27%	-22%	-16%	-30%	-25%	5%
TRI	6.32%	-1.3	3.98	-30%	-24%	-16%	-28%	-14%	32%
CM	6.43%	-0.61	1.12	-30%	-25%	-18%	-36%	-34%	-14%
BMO	6.46%	-0.09	1.2	-30%	-25%	-19%	-35%	-35%	-20%
SLF	6.65%	-0.42	2.7	-31%	-25%	-19%	-36%	-34%	-13%
XEG	6.78%	-0.54	1.17	-33%	-27%	-20%	-39%	-35%	-18%
IAG	6.93%	-0.59	3.14	-33%	-26%	-20%	-38%	-37%	-18%
T	7.16%	0.09	1.41	-33%	-28%	-21%	-41%	-42%	-36%
RCI.B	7.22%	0.19	0.44	-33%	-27%	-21%	-41%	-41%	-30%
ECA	7.65%	-0.38	0.41	-37%	-31%	-23%	-44%	-46%	-36%
TLM	7.85%	-0.56	0.53	-37%	-31%	-24%	-46%	-48%	-40%
XMA	8.51%	-0.82	2.96	-39%	-32%	-24%	-45%	-44%	-26%
SU	9.49%	-0.42	2.3	-44%	-36%	-27%	-55%	-57%	-51%
XGD	9.60%	0.19	0.93	-42%	-36%	-28%	-54%	-57%	-56%
MFC	9.68%	-0.26	7.83	-47%	-38%	-29%	-57%	-61%	-56%
CNQ	9.90%	0.03	-0.08	-43%	-38%	-30%	-57%	-63%	-62%
ABX	9.99%	0.32	1.63	-43%	-37%	-29%	-56%	-62%	-62%
POT	10.68%	0.26	2.42	-48%	-41%	-33%	-62%	-69%	-71%
G	12.03%	0.81	3.7	-49%	-42%	-34%	-65%	-72%	-77%
RIM	15.78%	0.3	0.83	-60%	-52%	-42%	-76%	-82%	-87%
TCK.B	17.54%	0.47	5.75	-69%	-60%	-48%	-82%	-88%	-92%

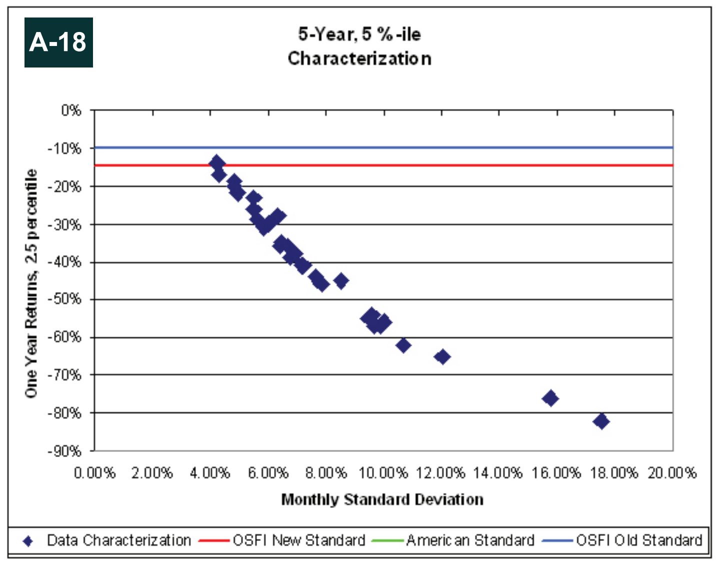
Characterization with a Security Template

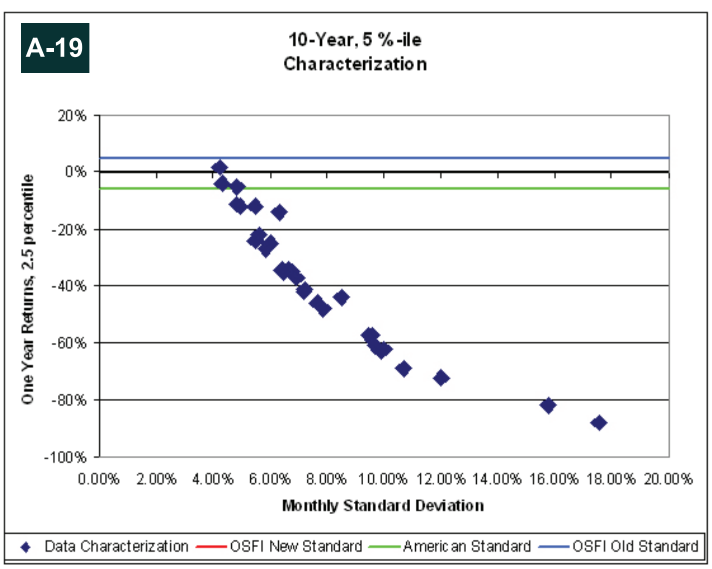
It is always desirable to use actual data, as opposed to mathematically derived ‘synthetic data’ for analysis and therefore a wide variety of securities were used, using their monthly returns for the period 2002-12-8 to 2010-12-8 as the basis for analysis. The results of these characterizations are reported on Table A-12 and charts relating these results to the standard deviation of the monthly returns are shown as Charts A-15 to A-20.

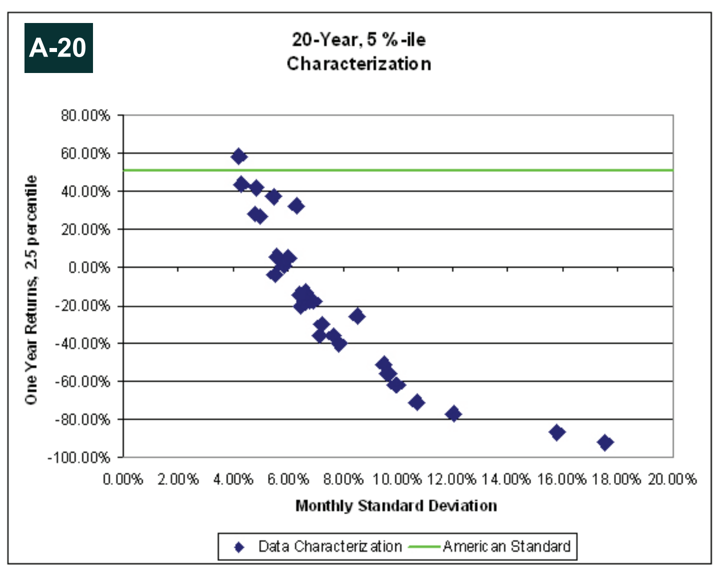

A-15
1-Year, 2.5 %-ile
Characterization
0%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
-80%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
OSFI New Standard
American Standard
OSFI Old Standard


A-16
1-Year, 5 %-ile
Characterization
0%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
OSFI New Standard
American Standard
OSFI Old Standard


A-17
1-Year, 10 %-ile
Characterization
0%
-10%
-20%
-30%
-40%
-50%
-60%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
OSFI New Standard
American Standard
OSFI Old Standard


A-18
5-Year, 5 %-ile
Characterization
0%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
-80%
-90%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
OSFI New Standard
American Standard
OSFI Old Standard


A-19
10-Year, 5 %-ile
Characterization
20%
0%
-20%
-40%
-60%
-80%
-100%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
OSFI New Standard
American Standard
OSFI Old Standard


A-20
20-Year, 5 %-ile
Characterization
80.00%
60.00%
40.00%
20.00%
0.00%
-20.00%
-40.00%
-60.00%
-80.00%
-100.00%
One Year Returns, 2.5 percentile
0.00% 2.00% 4.00% 6.00% 8.00% 10.00% 12.00% 14.00% 16.00% 18.00% 20.00%
Monthly Standard Deviation
Data Characterization
American Standard

As discussed in the section “Modified Latin Hypercube Method”, there will be no adjustments made to the raw results for the first five years of the simulation.

Accordingly, to choose a security template, we need an issue that meets the OSFI New Standard at the one-year and five-year marks, and is reasonably close to the standard for the ten-year mark. The best fit is ECA, with a monthly standard deviation of 7.65%. As may be seen from Chart A-12, the fifth percentile of the twenty-year returns for this issue is -36%, whereas the US actuarial standard allows this figure to be anywhere up to 51%, so the next step is to see what degree of mean-reversion can be applied to the raw results while still meeting all the standards.

As an aside, it is interesting to note that of the thirty securities tested, only twelve met the OSFI standard for (1-Year, 2.5%), sixteen met the (1-Year, 5%) standard and twenty-three met the (1-Year, 10%) standard; this is a reflection of OSFI’s propensity for regime-shifting models as it is clear that the overall distribution has a fatter-tail than is experienced even with live data over a period that includes the depths of the Credit Crunch.

Be that as it may, Table A-13 shows the results of changing the Decile Sampling Factor (DSF) for ECA when characterizing the raw data. As may be seen, we can achieve reasonably close approximations to the standards when the DSF is set to 1.30.

We have now achieved our objective of creating a simple model that is a reasonably close approximation to the more complex models used by insurance companies to estimate their capital requirements for guarantees on Segregated Funds and Variable Annuities. We may now proceed to use this model for a variety of retirement withdrawal calculations.

Table A-13: Effect of Changing DSF on 5%-ile value of Simulation Return Template ECA.TO, sample 2002-12-8 to 2010-12-8, 7% Expected Annual Return

Description	1 Year	5 Years	10 Years	20 Years
US Actuarial Maximum	**N/A**	**-19%**	**-6%**	**+51%**
OSFI New Standard	**-26%**	**N/A**	**N/A**	**N/A**
DSF = 0.01	-31%	-44%	-46%	-36%
DSF = 0.10	-30%	-45%	-42%	-23%
DSF = 0.20	-29%	-45%	-39%	-17%
DSF = 0.30	-30%	-45%	-38%	-15%
DSF = 0.40	-31%	-46%	-34%	-3%
DSF = 0.50	-30%	-46%	-29%	+4%
DSF = 0.60	-30%	-45%	-27%	+8%
DSF = 0.70	-31%	-45%	-22%	+19%
DSF = 0.80	-31%	-45%	-20%	+22%
DSF = 0.90	-30%	-44%	-18%	+29%
DSF = 1.00	-31%	-46%	-16%	+32%
DSF = 1.10	-30%	-45%	-12%	+37%
DSF = 1.20	-30%	-45%	-10%	+49%
DSF = 1.30	-31%	-45%	-9%	+48%
DSF = 1.40	-30%	-46%	-5%	+51%
DSF = 1.50	-31%	-44%	-4%	+57%

For convenience in reading, the less stringent standard has been reported as N/A

Results differ from those previously reported in Table SI-7 of the essay at http://www.himivest.com/media/SecurityIncome.pdf, published in the October, 2011, edition of this newsletter due to the correction of an error.

Equity Mean-Reversion and Retirement Withdrawal Success Rates

In my essay *Security of Income vs. Security of Principal*,[46] I compared the results of the Retirement Withdrawal Calculator with those of Bernstein,[47] which are shown in Table A-14.

Table A-14: 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5% (from Bernstein, 2001)

			Withdrawal Rate			
	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	12.0%	88.8%	70.4%	48.0%	28.3%
75/25	4.4%	9.09%	94.3%	75.2%	46.6%	22.2%
50/50	4.2%	6.5%	98.2%	80.1%	41.3%	12.0%
25/75	3.9%	4.8%	99.5%	81.1%	28.9%	3.4%
Bonds	3.5%	5.0%	98.2%	68.3%	18.5%	0.2%
Success is defined as reaching the thirty-year mark with a portfolio value that exceeds zero						

In an attempt to replicate these data, the Retirement Withdrawals Calculator was run with the parameterization shown in Table A-15.

Table A-15: HIMI Retirement Calculator Assumptions Used to Replicate Bernstein (2001)

Input Assumption	Value
Start Date	2002-12-8
End Date	2010-12-8
Symbol	*0350
Expected Annualized Return [Equities]	4.50%
Dividend Yield [Equities]	0.00%
Bonds Expected Return	3.50%
Bonds Coupon	0.00%
Standard Deviation	5.00%
Annuity Rate	630.00
Correlation	0.00
Inflation	0.00
Stocks, Percentage	Varied
Bonds, Percentage	Varied
Annuity, Percentage	0
Initial Withdrawal Rate	Varied

The objective of the exercise was to demonstrate the importance of cash income in retirement withdrawal planning, so two sets of results were prepared: one assuming that cash income from the investments was zero, and the second assuming more reasonable figures. Results of these simulations are shown in Tables A-16 and A-17.

[46] Available on-line at http://www.himivest.com/media/SecurityIncome.pdf, also included in October, 2011, edition of this newsletter.

[47] William J. Bernstein, *The Retirement Calculator From Hell – Part II*, 2001, available on-line at http://www.efficientfrontier.com/ef/101/hell101.htm (accessed 2012-4-8)

Table A-16: 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5% Zero Cash Income						
			Withdrawal Rate			
	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	12.0%	84.2% (-4.6)	63.6% (-6.8)	42.0% (-6.0)	23.2% (-5.1)
75/25	4.4%	9.09%	92.6% (-1.7)	69.5% (-5.7)	43.2% (-3.4)	17.0% (-5.2)
50/50	4.2%	6.5%	97.4% (-0.8)	75.6% (-4.5)	38.7% (-2.6)	11.0% (-1.0)
25/75	3.9%	4.8%	98.9% (-0.6)	79.1% (-2.0)	31.8% (+2.9)	4.8% (+1.4)
Bonds	3.5%	5.0%	98.4% (-0.2)	72.2% (+3.9)	24.5% (+6.0)	3.0% (+2.8)

Success is defined as reaching the thirty-year mark with a portfolio value that exceeds zero
Bracketted figures indicate difference from the Bernstein, 2001, results shown in Table A-14

Table A-17: 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5% Cash Income (Current Yield): Stocks 2.25%, Bonds 3.5%						
			Withdrawal Rate			
	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	12.0%	89.4% (+5.2)	71.8% (+8.2)	50.5% (+8.5)	30.8% (+7.6)
75/25	4.4%	9.09%	95.4% (+2.0)	78.6% (+9.1)	50.6% (+7.4)	24.8% (+7.8)
50/50	4.2%	6.5%	98.2% (+0.8)	83.1% (+7.5)	47.0% (+8.3)	15.3% (+4.3)
25/75	3.9%	4.8%	99.4% (+0.5)	84.0% (+4.9)	32.8% (+1.0)	5.6% (+0.8)
Bonds	3.5%	5.0%	97.5% (-0.9)	71.6% (-0.6)	23.1% (-1.4)	3.1% (+0.1)

Success is defined as reaching the thirty-year mark with a portfolio value that exceeds zero
Bracketted figures are the change from Table A-16, in which all returns are assumed to be price-based, with no regular income.
Results differ from those previously reported in Table SI-7 of the essay at http://www.himivest.com/media/SecurityIncome.pdf, published in the October, 2011, edition of this newsletter due to the correction of an error.

As may be seen, the incorporation of cash income as a relatively constant contribution to the portfolio (while keeping the total expected returns of the portfolio components constant) has a very dramatic effect and illustrates why bonds are useful for retirement portfolios: although the reduced standard deviation of returns helps to achieve portfolio goals, the crucial difference is that the entire return of bonds is received annually in cash; this reduces the necessity of selling investments at poor prices in order to fund a cash withdrawal, which is the source of Sequence of Returns Risk.

We may now examine the effects of the Decile Sampling Factor, which may be expected to improve the performance of portfolios with a heavy equity weighting; note that with a monthly standard deviation of 3.5% and DSF = 0.01, the 20-Year 5-percentile return is +8.30% and the 10-percentile is +30.14%; but with DSF=1.30, the comparable figures are +53.85% and +70.77%, respectively. These are significant changes, to say the least!

Table A-18: 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5% Cash Income (Current Yield): Stocks 2.25%, Bonds 3.5% Bell Curve Distribution DSF = 1.3						
			Withdrawal Rate			
	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	12.0%	99.9% (+10.5)	89.2% (+17.4)	51.7% (+1.2)	18.1% (-12.7)
75/25	4.4%	9.09%	99.9% (+4.5)	93.9% (+15.3)	52.2% (+1.6)	10.6% (-14.2)
50/50	4.2%	6.5%	100% (+1.8)	93.2% (+10.1)	43.8% (-3.2)	5.2% (-10.1)
25/75	3.9%	4.8%	100% (+0.6)	87.1% (+3.1)	31.3% (-1.5)	2.8% (-2.8)
Bonds	3.5%	5.0%	98% (+0.5)	70.2% (-1.4)	23.8% (+0.7)	4.2% (+1.1)

Success is defined as reaching the thirty-year mark with a portfolio value that exceeds zero
Bracketted figures are the change from Table A-17, in which all returns are assumed to be price-based, with no regular income.

The pattern of changes between Tables A-17 and A-18 (shown in brackets in Table A-18) is easy to understand intuitively: extreme values of distributions over the thirty years of simulation are discouraged when the Decile Sampling Factor forces a more even distribution of monthly returns. This reduces the chances of an extreme result, regardless of whether, given the simulation conditions, an extreme result is considered “success” or “failure”.

Table A-19: 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5%
Cash Income (Current Yield): Stocks 2.25%, Bonds 3.5%
Equity Template: ECA
DSF = 1.3

			Withdrawal Rate			
	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	28.2%	85.2% (-14.7)	71.4% (-17.8)	52.2% (+0.5)	36.8% (+18.7)
75/25	4.4%		95.5% (-4.4)	82.2% (-11.7)	59.4% (+7.2)	40.0% (+29.4)
50/50	4.2%		99.1% (-0.9)	89.6% (-3.6)	65.5% (+21.7)	35.0% (+29.8)
25/75	3.9%		99.8% (-0.2)	93.4% (+6.3)	53.8% (+22.5)	18.6% (+15.8)
Bonds	3.5%	5.0%	98.4% (+0.4)	69.6% (-0.6)	23.3% (-0.5)	3.1% (-1.1)

Success is defined as reaching the thirty-year mark with a portfolio value that exceeds zero
Bracketted figures are the change from Table A-19, which has a Standard Deviation of monthly equity returns of 12% in a Bell Curve.

The differences between Table A-18 and A-19 (shown in brackets in Table A-19) are also fairly easy to understand intuitively: the increase in dispersion of returns caused by the change in the equity return template have resulted in an increase in the likelihood of an extreme result, regardless of whether the extreme is considered to be ìsuccessî or ìfailureî given the conditions of the simulation.

Implied Longevity Yield: The Importance of Residual Value

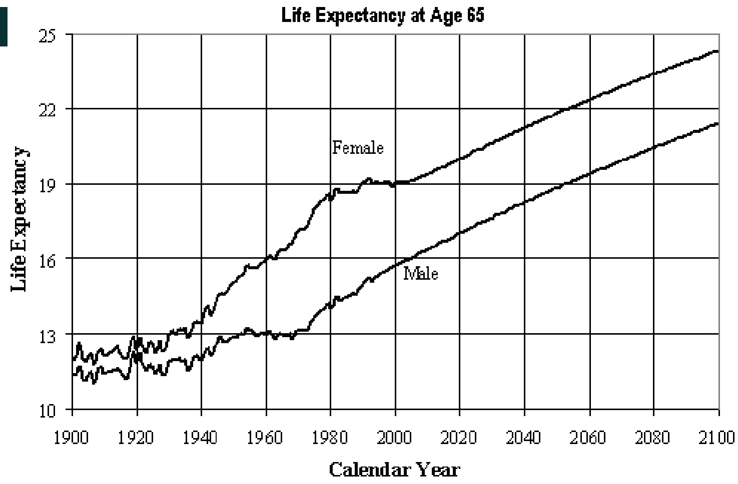
Dr. Moshe Milevsky developed[48] Implied Longevity Yield to *help promote and explain the benefits of acquiring lifetime payout annuities by translating the abstract-sounding longevity insurance into more concrete and measurable financial rates of return*. For example, using figures from 2003-11-26, Dr. Milevsky asks: *What would happen if the 65 year-old male decided to forgo the purchase of a life annuity and instead invested the $100,000 and withdrew the same exact $678.22 per month for the next 10 years? This strategy is called self-annuitization. What would be the required portfolio investment return needed to successfully withdraw $678.216 per month AND still have $69,396 at the end of ten years to purchase an identical annuity? This number is precisely the Implied Longevity Yield (ILY) at age 65.*

However, Dr. Milevsky does not consider bequest motives in his formulation; nor does he consider the potential for the individual to seek to accelerate consumption after the receipt of bad news. As a noted in the April, 2010, edition of this newsletter: *Dead and alive are not the only states and retirees do not, as a rule, drop dead immediately after drawing the losing ticket in the mortality pool. It is not unusual for a senior to be told of a degenerative disease; of impending blindness; or for other medical problems to be diagnosed that drastically alter his planning horizon. In these circumstances, the ability to shift consumption into the period in which it can be best enjoyed has immense value, and this change in consumption timing is not possible with an annuity*.

The calculation for Implied Longevity Yield contains the embedded assumption that the retiree will, in fact, live until the terminal date of the calculation; or, to put it another way, ascribes a value of zero to the residual cash in the event that the retiree dies during the period of self-annuitization while also assigning zero value to the potential for accelerating consumption during this period.

Life expectancy at age 65 has been increasing dramatically since 1970 (see Chart A-21),[49] but in 2007 the U.S. Social Security Administration reports[50] that of 79,684 males aged 65, only 61,612 would celebrate their 75th birthdays. Thus, for the general populace, the chance of death in that ten year span is about 23% – so setting the chance to zero, as is implicitly done by the ILY calculation, will result in a much higher required rate of return (and therefore, a greater benefit that may be derived immediate annuitization) than would be the case if the probability of death were to be taken into account.

A-21



[48] Moshe A. Milevsky, *The Implied Longevity Yield: A Note on Developing an Index for Life Annuities*, 2004-9-15, available on-line at http://www.ifid.ca/pdf_workingpapers/WP_2004Sept15.pdf (accessed 2012-4-8)

[49] Felicitie C. Bell, Michael L. Miller, *Life Tables for the United States Social Security Area 1900–2100*, U.S. Social Security Administration, Actuarial Study No. 116, available on-line at http://www.ssa.gov/oact/NOTES/as116/as116_V.html#wp998997 (accessed 2012-4-8)

[50] *Period Life Table 2007*, U.S. Social Security Administration, available on-line at http://www.socialsecurity.gov/OACT/STATS/table4c6.html (accessed 2012-4-8)

In short, there are three major problems with the concept of Implied Longevity Yield:

- The value of leaving an estate in the interim is set to zero
- The value of being able to accelerate consumption in the interim is set to zero
- The probability of interim high inflation is set to zero.

Taxation of Annuities[51]

Prescribed:

Prescribed annuities are generally preferred by investors and advisors as they are taxed on a preferential basis,but this preference is accompanied by the requirement that they be purchased with non-registered funds, among other conditions. The taxable portion of a prescribed annuity is:

Taxable Income = Annual payment – Single premium/Life Expectancy

So, for example, if a client pays $100,000 for an annuity paying $8,460 annually and has a life expectancy of 13.76 years, the taxable income is calculated as

Annual Taxable Income = 8,460 – 100,000/13.76 = $1,192

The remaining portion of the annual payments, 8,460 – 1,192 = 7,268, is considered (quite rightly!) to be return of capital and is not taxed – even if the annuitant should live longer than expected and hence have more capital returned than he originally deposited. A nice little bonus with this treatment is that the government uses the 1971 mortality tables for this calculation; the slightly shorter life expectancy increases the return-of-capital amount to above what might be expected today, decreasing the taxable amount.

Leveraged ETFs: An example of Cash Flow Effects

A reader brought a blog post[52] to my attention that discussed the difference between arithmetic and geometric returns. This post referred to a more formal paper by Tony Cooper,[53] which is notable for having a complete misunderstanding of the mechanics of leveraged ETF management and blames the well-known drag on leveraged ETF returns on volatility by itself.

The SEC has a much better explanation,[54] but does not make the mechanism of the underperformance sufficiently clear: *Here's a hypothetical example: let's say that on Day 1, an index starts with a value of 100 and a leveraged ETF that seeks to double the return of the index starts at $100. If the index drops by 10 points on Day 1, it has a 10 percent loss and a resulting value of 90. Assuming it achieved its stated objective, the leveraged ETF would therefore drop 20 percent on that day and have an ending value of $80. On Day 2, if the index rises 10 percent, the index value increases to 99. For the ETF, its value for Day 2 would rise by 20 percent, which means the ETF would have a value of $96. On both days, the leveraged ETF did exactly what it was supposed to do – it produced daily returns that were two times the daily index returns. But let's look at the results over the 2 day period: the index lost 1 percent (it fell from 100 to 99) while the 2x leveraged ETF lost 4 percent (it fell from $100 to $96). That means that over the two day period, the ETF's negative returns were 4 times as much as the two-day return of the index instead of 2 times the return.*

The SEC is quite right with this example but, although referring to portfolio resets for leveraged ETFs they do not provide a numerical example of the manner in which these resets work.

Scenario 1

Let us assume that the leveraged ETF has an initial value of $100 held in cash, and seeks to double the daily return of the index ETF (IETF). It accomplishes this by buying two units of IETF at $100, using its $100 cash and borrowing $100. To follow the SEC example, the price of IETF immediately plummets 10% to $90, so the balance sheet of LETF is now as shown in Table A-6:

Table A-6: LETF Balance Sheet, Scenario 1, Day 1, Before Reset

Assets		Liabilities & Equity	
2.00 IETF Units @ $90	$180	Borrowing	$100
		LETF Equity	$80

[51] From the April, 2010, edition of this newsletter

[52] David Varadi, *Understanding the Link Between Volatility and Compound Returns*, CSS Analytics, 2012-3-12, available on-line at http://cssanalytics.wordpress.com/2012/03/12/understanding-the-link-between-volatility-and-compound-returns/ (accessed 2012-3-24)

[53] Tony Cooper, *Alpha Generation and Risk Smoothing Using Managed Volatility*, Double-Digit Numerics, 2010-8-6, available on-line via http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1664823 (accessed 2012-3-24)

[54] U.S. Securities and Exchange Commission, *Leveraged and Inverse ETFs: Specialized Products with Extra Risks for Buy-and-Hold Investors*, 2009-8-18, available on-line at http://www.sec.gov/investor/pubs/leveragedetfs-alert.htm (accessed 2012-3-24)

It is plain to see from Table A-6 that LETF is now over-leveraged: it owns $180 of the index, but only has $80 in equity. In order to reset its portfolio so that it can achieve its objective of delivering 2x index returns on each day, it must sell some of its IETF holdings, to arrive at Table A-7:

Table A-7: LETF Balance Sheet, Scenario 1, Day 1, After Reset			
Assets		**Liabilities & Equity**	
1.78 IETF Units @ $90	$160	Borrowing	$80
		LETF Equity	$80

On Day 2 of the SEC Scenario, the index rises 10%, resulting in the value of its holdings rising to $176, as shown in Table A-8:

Table A-8: LETF Balance Sheet, Scenario 1, Day 2, Before Reset			
Assets		**Liabilities & Equity**	
1.78 IETF Units @$99	$176	Borrowing	$80
		LETF Equity	$96

As the SEC indicates, the unitholder equity in the LETF is now $96, a decrease of 4% from the initial level, despite the fact that IETF is down only 1%. However, before summarizing, let us examine Scenario 2.

Scenario 2

The leveraged ETF again has an initial value of $100 held in cash, and seeks to double the daily return of the index ETF (IETF). It accomplishes this by buying two units of IETF at $100, using its $100 cash and borrowing $100. However, this time the value of IETF increases 10% by the end of the day, resulting in the summary balance sheet of Table A-9:

Table A-9: LETF Balance Sheet, Scenario 2, Day 1, Before Reset			
Assets		**Liabilities & Equity**	
2.00 IETF Units @$110	$220	Borrowing	$100
		LETF Equity	$120

Table A5 shows that LETF is now under-leveraged: it owns $220 of the index, but has $120 in equity. In order to reset its portfolio so that it can achieve its objective of delivering 2x index returns on each day, it must buy additional IETF units, to arrive at Table A-10:

Table A-10: LETF Balance Sheet, Scenario 2, Day 1, After Reset			
Assets		**Liabilities & Equity**	
2.18 IETF Units @$110	$240	Borrowing	$120
		LETF Equity	$120

On Day 2 of the Scenario 2, the index declines 10%, resulting in the summary balance sheet of Table A-11:

Table A-11: LETF Balance Sheet, Scenario 1, Day 2, Before Reset			
Assets		**Liabilities & Equity**	
2.18 IETF Units @99	$216	Borrowing	$120
		LETF Equity	$96

In Scenario 2, therefore, the LETF has again declined by 4% in value, despite the fact that the index has again declined by only 1%.

Explanation

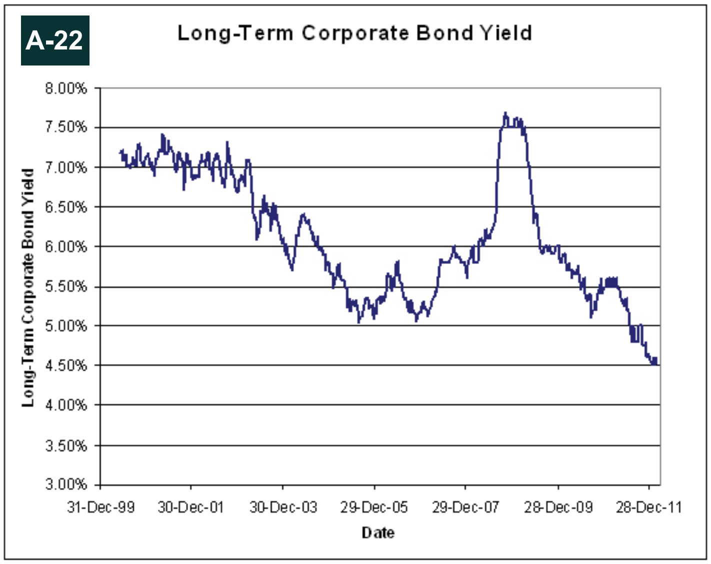
The volatility drag explains the fact that the final value for IETF is $99, after having experienced returns of +10% and -10%.

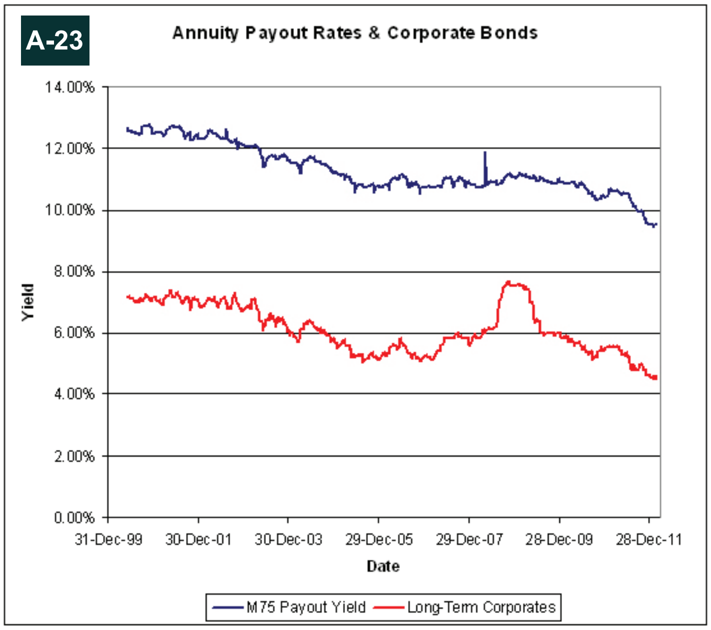
However, it is the market action by LETF that has magnified this loss. The leveraged fund has bought units as the price has increased (it must, because its equity will, by design, increase at a faster rate than the rate of increase in the underlying IETF) and it has sold units as the price decreased.

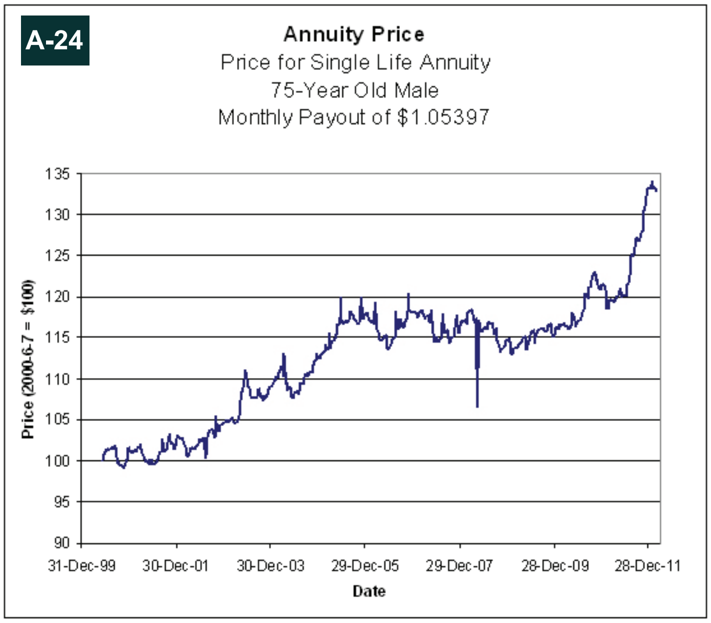
Thus, the LETF has introduced sequence of returns risk into its portfolio, due to its avowed practice of buying when prices are high and selling when prices are low – or, to put it another way, the effect of cash flows on a portfolio in a particularly nasty and deterministic manner.

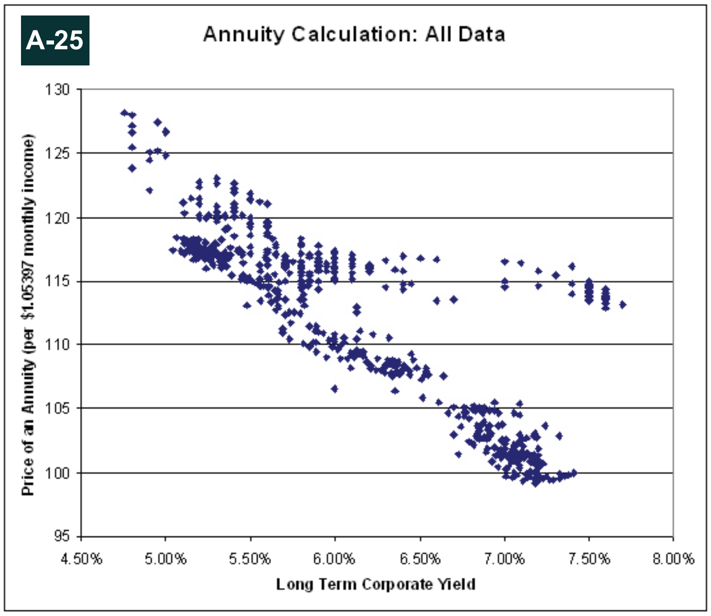
Annuities[55]

We are living in interesting times! As shown in Chart A-22, long-term corporate bond yields have not just recovered from their spike during the Credit Crunch, but they are now plumbing new depths. This has been reflected in the Payout Yield of Annuities (it will be remembered that the Payout Yield incorporates return of capital; not just the annuitants capital, but the capital of other clients of the insurer who have expired prior to the actuarially assumed age of death[56]) which has declined dramatically, with the Payout Yield for a seventy-five year old male now below 10%, as shown in Chart A-23. To put it another way, the price of an annuity has spike over the past year, as shown in Chart A-24.









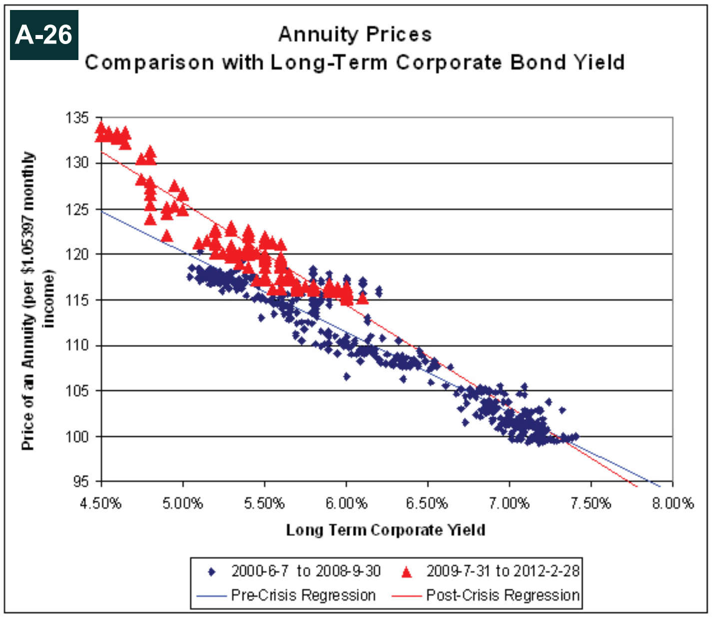
55 Annuity data used to prepare this section has been taken from the IFID Annuity Payout Index, available on-line at http://ifid.ca/payout.htm (accessed 2012-4-15)

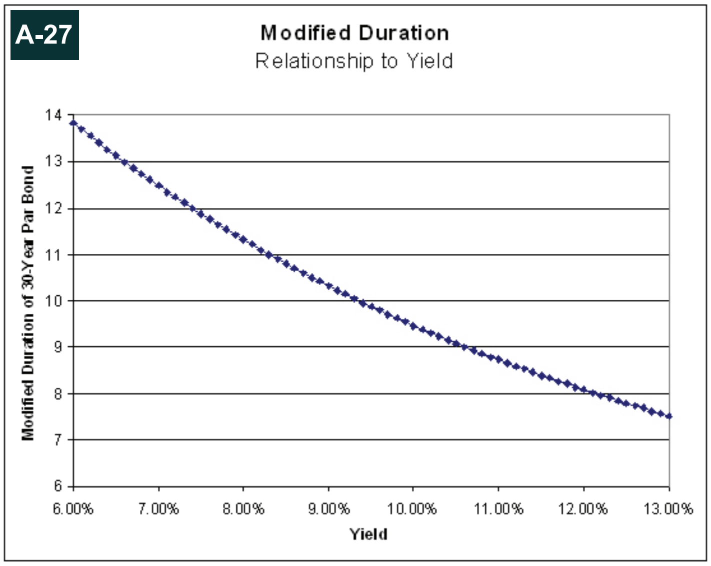
56 This is a simplification, and presents annuities as a tontine type – a tontine being, essentially, a closed group, funded at inception by a group, and paying income only to survivors. In fact, insurance companies price annuities according to statistical mortality tables, and assume the risk that the lives of their clients will extend further than their assumptions.

As has been discussed in the April, 2010, and April, 2011, editions of this newsletter, the Credit Crunch represented a watershed in the pricing of annuities. It is clear from Chart A-23 that the spike in long-term corporate yields during this period was not matched by a similar spike in annuity pay-out yields. This could result from several factors – a reluctance by insurers to write new business during the crisis due to their own problems with capital and regulation, the use of a more sophisticated model taking into account the unlikelihood of long term corporate yields lasting for any extended period of time, lack of availability of corporate bonds in the marketplace that would allow insurers to match their assets with liabilities or, possibly, a simple business decision that there was no need to increase payout rates because they were getting all the business they could handle without it, due to panic-stricken seniors fleeing the stock market and buying annuities at whatever price they were given. It would certainly be interesting to know which, if any, of these possibilities was most applicable!

One way or another, the Credit Crunch messed up the relationship between long-term corporate yields and payout yields, resulting in the rather odd Y-shaped pattern displayed in Chart A-25.

However, when the data is disaggregated into pre-crisis and post-crisis relationships, the strength of the correlation is clear, as shown in Chart A-26. The two correlations are quite strong, with an adjusted-R-squared of 92% pre-crisis and 88% post crisis. It is of interest to note that post-crisis annuities are more expensive than pre-crisis annuities when long-term corporate bond yields are held constant – this could reflect increased capital costs due to regulation for the insurers, or it could simply reflect an opportunistic increase in margins. Also of interest is the slope of the correlation lines: this slope represents the Modified Duration[57] of annuity prices, which was 8.9 pre-crisis but has now increased to 11.2 post-crisis. However, it is not clear whether this apparent increase in sensitivity is real, or is simply due to convexity – the change of Modified Duration with yield that is an integral part of simple yield theory. This effect is illustrated in Chart A-27.





In connection with Modified Duration, it will be noted that those wishing to hedge an eventual purchase of an annuity (in this case, a single-life annuity purchased by a 75-year-old man) should ensure that the Modified Duration of their hedging portfolio matches the duration of the annuity: Two products that will assist in this regard are the BMO Mid Corporate Bond Index ETF,[58] duration 5.84, and the BMO Long Corporate Bond Index ETF,[59] duration 12.85. If these products are combined so that the value invested in long corporates is about three times the value invested in mid-corporates, the average duration will be about 11.2, equal to the current estimate of the annuity's Modified Duration. This is a change from the 50-50 split that was implied by the calculations in the April, 2011, edition of this newsletter but is, I suggest, more reasonable given the current life expectancy of the seventy-five year old man of just under eleven years.[60]

Naturally, given a base portfolio as described, investors may then seek to optimize their holdings through the use of other fixed income products, such as individual bonds held directly and preferred shares.

57 For more information on Modified Duration, see my article with that title on-line at http://www.himivest.com/media/moneysaver_0705.pdf

58 See http://etfs.bmo.com/bmo-etfs/glance?fundId=75744 (accessed 2012-4-15)

59 See http://etfs.bmo.com/bmo-etfs/glance?fundId=75747 (accessed 2012-4-15)

60 *Period Life Tables, 2007*, supra

Investment Conclusions

This has been a very long-winded and rambling essay, for which I apologize; but the subject of retirement withdrawals covers many areas of investment theory and I'm a long-winded person anyway.

The Retirement Withdrawals Calculator (available on-line at http://www.prefblog.com/xls/retirementWithdrawals_2012.xls) has been improved to account for the tendency of long-term equity returns to revert to the mean – or, if you prefer, for periods of high volatility to alternate with periods of low volatility. This may be used as a tool to assist in retirement portfolio planning – the role of cash returns is emphasized, as these payments will reduce the need to sell into the market in order to meet consumption requirements.

I regret that the calculator has not been enhanced to reflect differential taxation of dividends, income and capital gains – I simply ran out of time again!