Part II

In the July, 2011, edition of this newsletter I reviewed some of the more basic concepts surrounding yield calculations, namely Current Yield (CY), Yield to Maturity (YTM) and Yield to Worst (YTW).

In this issue, I will examine the faults of these measures more closely and introduce some of the measures utilized in HIMIPref™: Portfolio Yield, Cost Yield and Curve Yield and examine how each of these measures compares when used as a predictor of future returns.

Portfolio Yield

As has previously been discussed, the calculation of Yield-to-Worst (YTW) involves examining every scenario – barring default and assuming no change in market yields – for the future of the individual issue and selecting the scenario that results in the lowest yield to maturity (or yield to call) as being applicable. Thus, YTW assigns a 100% probability to this single scenario and, of course, a 0% chance to any other scenario. This is in distinction to Current Yield, which assigns a 100% probability to the scenario of perpetual existence of the instrument – even when perpetual existence is contrary to the terms of the issue.

When calculating Portfolio Yield, the following steps are taken:

- A schedule of potential maturity dates is created, each having a probability of exercise in excess of 5%
- The Yield-to-Call and Term of each of these scenarios is calculated
- A minimum term for the yield calculation[1] is chosen, essentially arbitrarily. HIMIPref™ uses a minimum term equal to 1.00.
- If the Term is less than the minimum term, then the "Restricted Yield" is equal to the Yield-to-Call times the proportion of the actual term to the minimum term. Otherwise, the Restricted Yield is equal to the Yield-to-Call
- The Portfolio Yield is the weighted (according to the scenario's probability) average of the Restricted Yields

The methodology is best illustrated by Table Y-1.

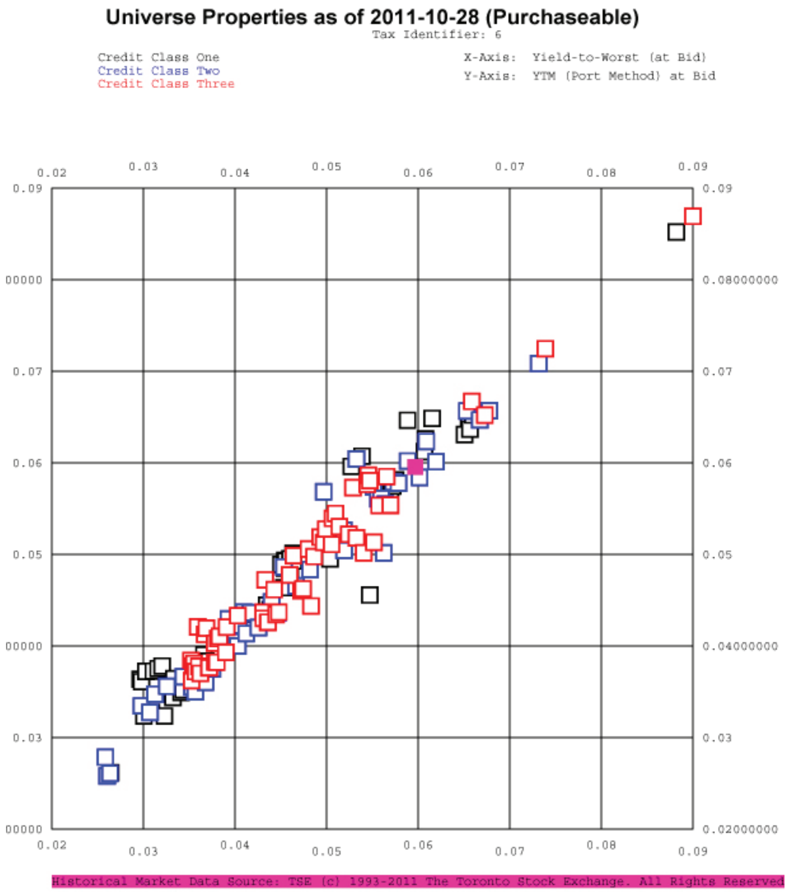
A graph of Portfolio Yield vs. Yield-to-Worst is displayed in Chart Y-1 and as may be seen there is good agreement between the two values with, as might be expected, a tendency for Portfolio Yield to be a little higher. An outlying point at about (5.5%, 4.5%) is PWF.PR.F, which is priced at 24.75 and has a cumulative probability of about 28% of being called within the next year; this means that the Restricted Yields become much more important in the calculation.

Table Y-1: Portfolio Yield Calculations for CM.PR.D (pays 1.4375, bid price 25.05) as of 2011-10-28

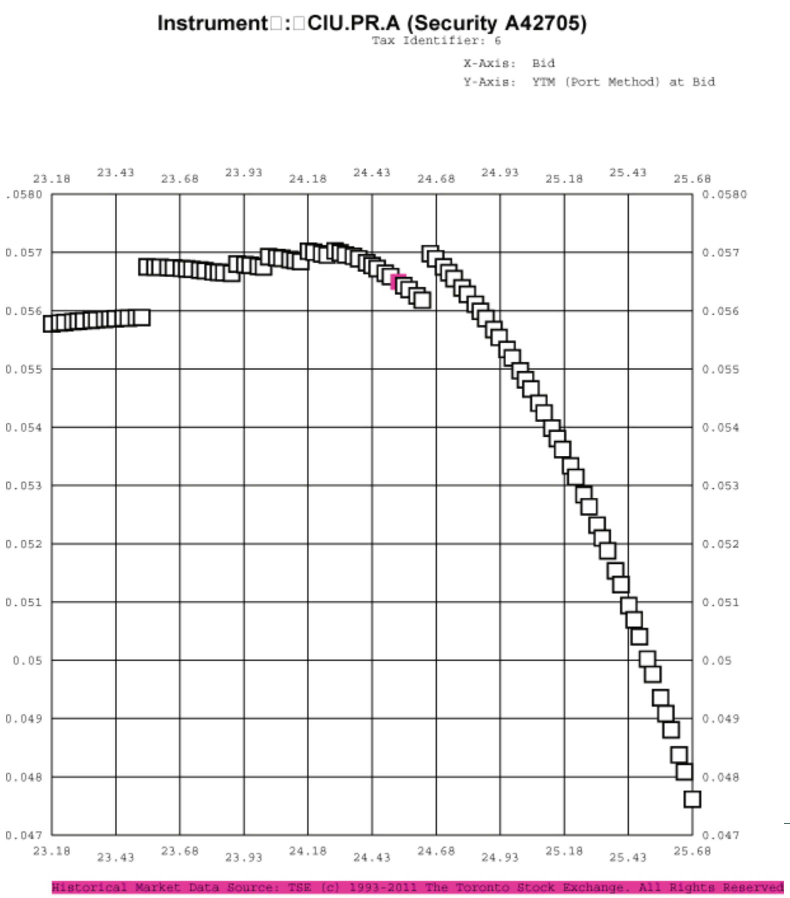
Call Date	Probability	Call Price	Yield to Call	Restricted Yield
2011-11-27	12.19%	25.25	15.36%	1.26%
2011-12-27	5.77%	25.25	10.49%	1.72%
2012-2-25	5.51%	25.25	8.02%	2.64%
2012-4-30	8.90%	25.00	5.23%	2.65%
2013-5-25	5.11%	25.00	5.62%	5.62%
2041-10-28*	62.51%	24.81	5.76%	5.76%
Portfolio Yield				**4.52%**
Note: Yield-to-Worst				**5.23%**
Note: Current Yield				**5.74%**
The last entry, with an end-date of 2041-10-28, is the LimitMaturity. The assumed end-price is 24.81.				

[1] This is a "system parameter" with the identifier PARAMETER_SYSTEM_PORTYIELDMINTERM

Y-1



Y-2



The concept of Restricted Yield was introduced during the programming of HIMIPref™ when it became apparent that the value of the Portfolio Yield calculation was being compromised by very high short-term yields to call. In effect, the probability of exercise for short-term redemptions declined more slowly than the increase in yield as the difference between the call price and the current price increased. Thus, the restriction was developed that, effectively, applies a minimum term-to-call of one year for yield calculation purposes, while retaining the probability derived for the shorter, actual, period.

I will be the first to agree that the adjustment is arbitrary and intellectually unsatisfying – but all quantitative systems have a little bat's blood[2] in them somewhere. However, there is a further problem with the concept that becomes apparent when Portfolio Yields are calculated over a range of prices and a calculation of Pseudo-Modified Duration (Portfolio Yield) is attempted (See Box Y-1 for an explanation of this calculation). Chart Y-2 shows the calculated Portfolio Yield for CIU.PR.A on October 31 in a range centred on its bid price of 24.43, while Chart Y-3 shows the resultant Pseudo-Modified Duration.

2 An ingredient used that makes the potion more effective, without any apparent reason.

As may be seen from Chart Y-2, there are discontinuities in the calculated Portfolio Yield; these discontinuities are due to the HIMIPref™ practice of incorporating potential calls into projections only when the probability of exercise of the call equals or exceeds 5%. Table Y-2 shows the difference in calculation results when the price is varied from 24.58 to 24.61. As may be seen the increment of three cents in price has pushed the probability of call on 2013-6-1 (when the call price for the issue changes from 26.00 to 25.75) to just over the 5% probability minimum; this has accordingly been incorporated in the calculation schedule at the expense of the probability of call one year later at 25.50. This increase, representing a roughly 5% chance of a roughly 200bp increase in realized yield, is responsible for the increase in calculated Portfolio Yield accompanying an increase in price.

Table Y-2: Two Calculations of Portfolio Yield for CIU.PR.A as of 2011-10-31

Price = 24.58				Price = 24.61			
Call Date	**Yield to Call**	**Restricted Yield**	**Probability**	**Call Date**	**Yield to Call**	**Restricted Yield**	**Probability**
2012-6-1	15.99%	9.37%	8.14%	2012-6-1	15.77%	9.24%	8.46%
2013-1-27	9.89%	9.89%	5.49%	2012-12-28	10.15%	10.15%	5.02%
2014-6-1	6.40%	6.40%	11.13%	2013-6-1	8.05%	8.05%	5.31%
				2014-6-1	6.35%	6.35%	6.37%
2016-6-1	5.24%	5.24%	9.04%	2016-6-1	5.21%	5.21%	9.05%
2041-10-31	4.72%	4.72%	66.20%	2041-10-31	4.72%	4.72%	65.79%
Portfolio Yield		**5.62%**		**Portfolio Yield**		**5.70%**	

Box Y-1: Pseudo-Modified Duration

As we all know, Modified Duration represents the sensitivity of a fixed-income instrument's price with respect to its yield, as shown in what I like to call the Fundamental Equation of Fixed Income:[3]

$$\Delta P = - D_{MOD} \bullet \Delta I \qquad \text{(Equation 1)}$$

Where ΔP is the percentage change in price
D_{MOD} is the Modified Duration
ΔI is the change in percentage yield

In order to calculate the Modified Duration, D_{Mod}, we first need to calculate the Macaulay Duration, D_{Mac}, since:

$$D_{Mod} = D_{Mac}/(1 + (y/f)) \qquad \text{(Equation 2)}$$

Where: D_{Mod} is the Modified Duration
D_{Mac} is the Macaulay Duration
y is the yield-to-maturity of the instrument
f is the number of payments per year.

And

$$D_{Mac} = (\sum PV_i \bullet T_i) / (\sum PV_i) \qquad \text{(Equation 3)}$$

Where : PV_i is the Present Value of the i'th cash flow
T_i the time the i'th cash flow is received
i is just a counter that identifies each one of our cash flows

And finally, Present Value is the amount of money that, if received right now and invested at the yield of the instrument ("y", in Equation 2) would be equal to amount required at the required time ("T", in Equation 3).

All in all, we may think of Modified Duration as being the derivative of the Price-Yield equation or, in other words, if we were to draw a graph relating price to yield, then Modified Duration is slope of the line tangent to that curve.

All the above, however, is specific to option-free bonds and the certainty of the sequence of cash flows that lie at the foundation of Equation 3 is required in order to arrive at a closed-form solution for Modified Duration.

[3] The following mathematics is taken from my article *Modified Duration*, Canadian Moneysaver, May 2007, available on-line at http://www.himivest.com/media/moneysaver_0705.pdf

When the bond has options, no such closed-form solution is possible; not one of which I am aware of, at any rate! Thus, HIMIPref™ calculates a measure referred to as Pseudo-Modified Duration, which is intended to serve the same purpose as Modified Duration with respect to Equation 1.

To calculate Pseudo-Modified Duration, the software first calculates three yields corresponding to three equally spaced prices. Then the Percent Price Difference, ∂P, and the corresponding Yield Difference, ∂Y, are calculated by:

$$\partial P = (P_{HIGH} - P_{LOW}) / P_{MID}$$

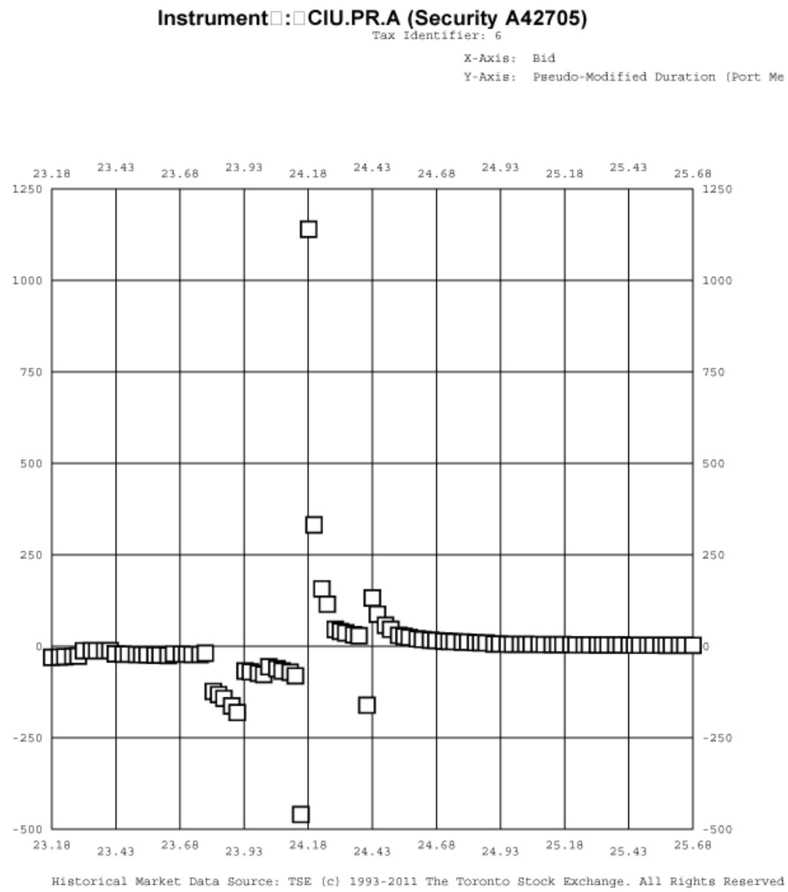
$$\partial Y = (Y_{HIGH} - Y_{LOW}) / Y_{MID}$$

$$PseudoModifiedDuration = - \partial P / \partial Y$$

The Price-Yield pairs are sampled with price intervals of 1% relative to P_{MID}. It will be noted that the PseudoModifiedDuration thus calculated will in fact approach the slope of the tangent line as the price interval approaches 0; but a relatively large price interval was chosen during programme design in order to reduce sudden changes and capture relatively likely option events.

An obvious method of mitigating the problem would be to decrease the minimum probability required for any particular call to be incorporated in the schedule. Such a reduction would, however, be computationally very expensive as more scenarios would have to be recorded throughout the calculation of all instruments in the universe of issues examined.

Y-3



As may be seen from Chart Y-3, the discontinuities in the calculation of PortfolioYield give rise to unpredictable behaviour of the corresponding PseudoModifedDuration, the calculation of which is described in Box Y-1.

As will be seen later, in the section in which various definitions of yield are used in an attempt to predict relative performance over a one-year period, PortfolioYield does have some value; that is, the idea of considering the yield of an instrument as the probability-weighted average of the yields-to-call does have some merit. In practical terms, however:

- Very high calculated yields for near term, low-probability calls require arbitrary adjustment if the results are to make any sense
- Using a high probability threshold for the incorporation of a scenario in the analysis results in discontinuous data; using a lower probability threshold becomes computationally very expensive.

The concept of costYield was developed in order to address these shortcomings.

CostYield

In the development of PortfolioYield, it was assumed that every potential call (exceeding the threshold value) contributed proportionately to a portfolio of instruments with a well-defined set of cash flows.

Another method of incorporating the embedded options into the analysis is to consider the case in which the investor first assumes that the cash flows received will be those that would be expected in the absence of options, and then adjusts these cash flows to reflect the effects on this expected schedule to reflect the options.

To illustrate, we may look again at the calculations for CIU.PR.A as of 2011-10-31. A cash-flow discounting table is prepared with:

- Bid price of -24.39 (negative by convention, so that the Present Value of the entire cash flow stream will equal zero) dated on the calculation date, 2011-10-31
- Quarterly Dividends of 0.2875, commencing 2011-12-1 and ending with the 2041-91 payment
- A “final dividend” of 0.19, payable on the LimitMaturity date of 2041-10-31
- An end price of 23.90 on the LimitMaturity date of 2041-10-31
- Option Effect entries as shown in Table Y-3.

Table Y-3: Option Effects for CIU.PR.A, Calculation Date 2011-10-31, Bid Price 24.39

Date	Probability of Exercise	Exercise Price	Unconstrained Price	Future Value
2012-6-1	6.33%	26.00	27.41	-0.09
2013-1-27	5.16%	26.00	28.40	-0.12
2014-6-1	10.73%	25.50	28.57	-0.33
2016-6-1	9.00%	25.00	29.34	-0.39

In Table Y-3:

- The probability of exercise is determined by a Black-Scholes option model
- The exercise price is from the issue’s call schedule
- Entries are incorporated in the table only when their probability exceeds 5%
- The unconstrained price is the weighted average price of the portion of the normal distribution in which the issue is expected to be called
- The future value is equal to the Probability of Exercise times the difference between the Unconstrained Price and the Exercise Price (i.e., the amount an investor can expect to lose, relative to the unconstrained price, if the issue is called)

It will be noted that the end-price of 23.90 is derived by determining the weighted average price if the last option is not exercised. It is also important to note that for this issue, the Yield-to-Worst is equal to the yield calculated to the limit maturity, for which the cash flow discounting table differs only in that for YTW, the entries of Table Y-3 are not incorporated (i.e., Cost Yield Cash Flows = Limit Maturity Cash Flows + Table Y-3).

Given this difference between the yield calculation to the LimitMaturity and the calculation of CostYield, it will be apparent that CostYield will always be the lower of the two figures. For this calculation, the CostYield is calculated to be 4.55%, while the yield to the LimitMaturity (which is, in this case, the Yield-to-Worst) is 4.76%.

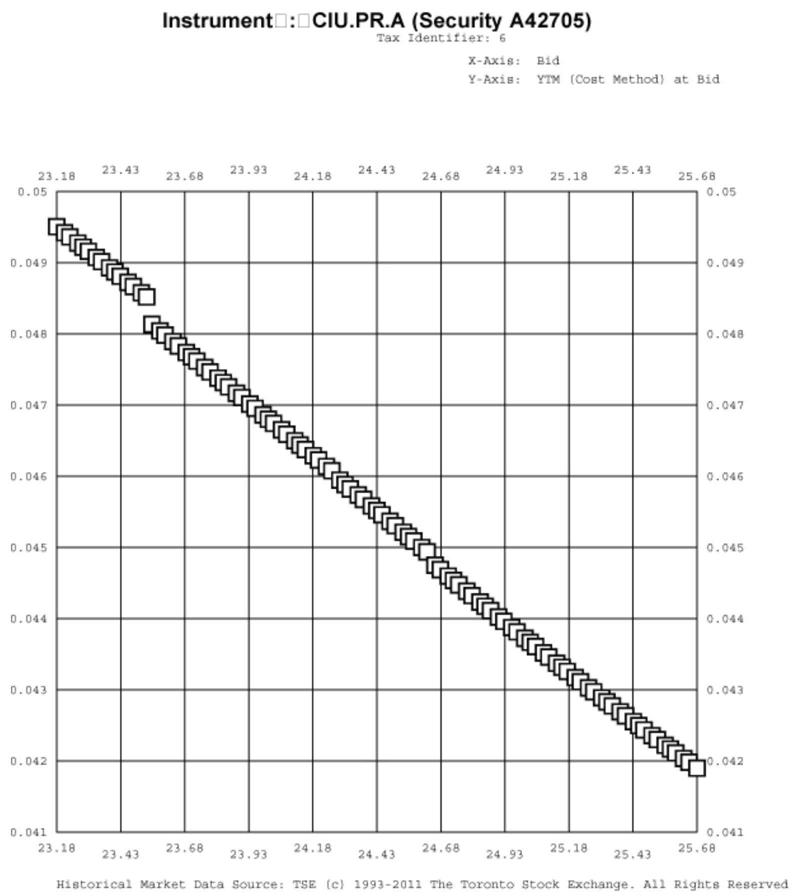
As readers will have come to expect, there are problems with the assumptions and calculation of CostYield:

- The price used as the input for each successive option calculation is the current price of 24.39, which embeds the assumption that the price on each option date is independent. One could make the argument that since each option calculation is dependent upon previous options not having been exercised, then the input price should reflect this:
    - E.g., in Table Y-3, the unconstrained price for the highest 6.33% of values calculated with a normal distribution for 2012-6-1 is 27.41, as shown. However, the weighted average price for the lowest 93.67% is 24.19. It would be perfectly reasonable to use this price as the expected future price for purposes of calculating probability for the next potential option, rather than the 24.39 current price that is currently used.
    - However, using this argument carries with it some embedded assumptions of its own:
        - Firstly, that the path of price over time retains its rank at all times within the universe of all possible paths; that is, if a particular path is at the 99th percentile at time X, then it will also be at the 99th percentile at time X+t. This is not consistent with the random evolution of prices embodied in Black-Scholes theory; in reality, there will be a certain amount of “cross-over”.
        - Secondly, Black-Scholes option theory has been used for all successive calculations, which necessitates an assumption of a normal distribution of possible prices on each option date. If one uses 24.19, the price derived from a truncated normal distribution, then it is unreasonable to expect that future distributions will not reflect the truncation. The assumption of future normal distributions arising from current non-normal distributions was shown[4] to be a critical factor in the faulty evaluation of the creditworthiness of tranches of Mezzanine Asset Backed Securities Collateral Debt Obligations that became apparent during the Panic of 2007.

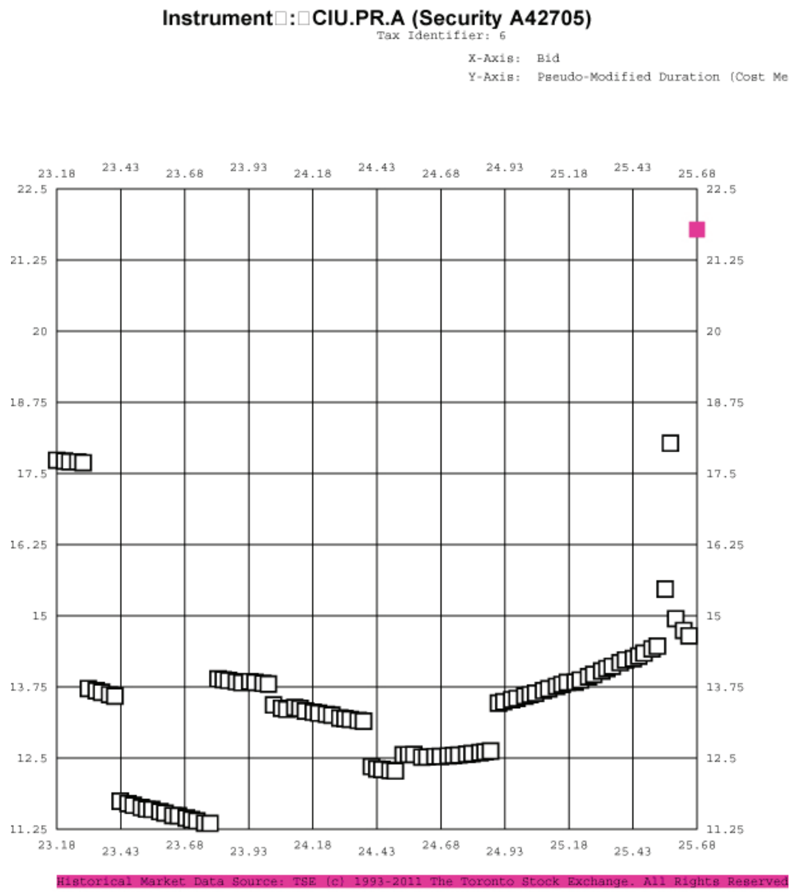
[4] John Hull and Alan White, *The Risk of Tranches Created from Mortgages*, May 2010, available on-line at http://www.rotman.utoronto.ca/~hull/downloadablepublications/AAArisk.pdf (accessed 2011-11-8)

- The minimum probability requirement for incorporation of a potential call has again resulted in discontinuities in the Price-Yield relationship (Chart Y-4) which has again resulted in strange and unrealistic behaviour for the calculated Pseudo-Modified-Duration as the price changes (Chart Y-5)
- There is a further problem with the computation of future prices, in that the assumption of normal distributions centered on the current price does not take into account the fact that the market price of any given instrument may be expected to reflect market knowledge of the potential for a call in the near future.
    - For example, we may have an issue with a very high coupon, but a potential call in the very near future. Such an issue will likely be trading very closely to its call price.
    - However, calculations dependent upon the market price will assume that there is a high probability that the issue will not be called, and that the high coupon may be enjoyed for some time in the future. This is not realistic.
    - For example Chart Y-6 shows the relationship between CostYield and YTW for a set of issues that is rated Pfd-1(low) or better and has duration in excess of a relatively arbitrary threshold. As may be seen there is good agreement between the two values, but there is a cluster of issues with a YTW of about 3% and a CostYield slightly in excess of 4%. These issues are all high-coupon FixedResets. Cost Yield assumes that since these issues are trading relatively close to their call price, there is a good chance they will not be called; whereas in fact they are trading relatively close to their call price because the chance of them being called is very high.
    - Chart Y-7 shows a superset of the selection shown in Chart Y-6; this set includes all instruments in the HIMIPref™ universe with a rating of Pfd-1(low) or better, with no duration threshold. As may be seen, the addition of these low-duration instruments substantially increases the proportion of instruments for which CostYield is significantly in excess of YTW.

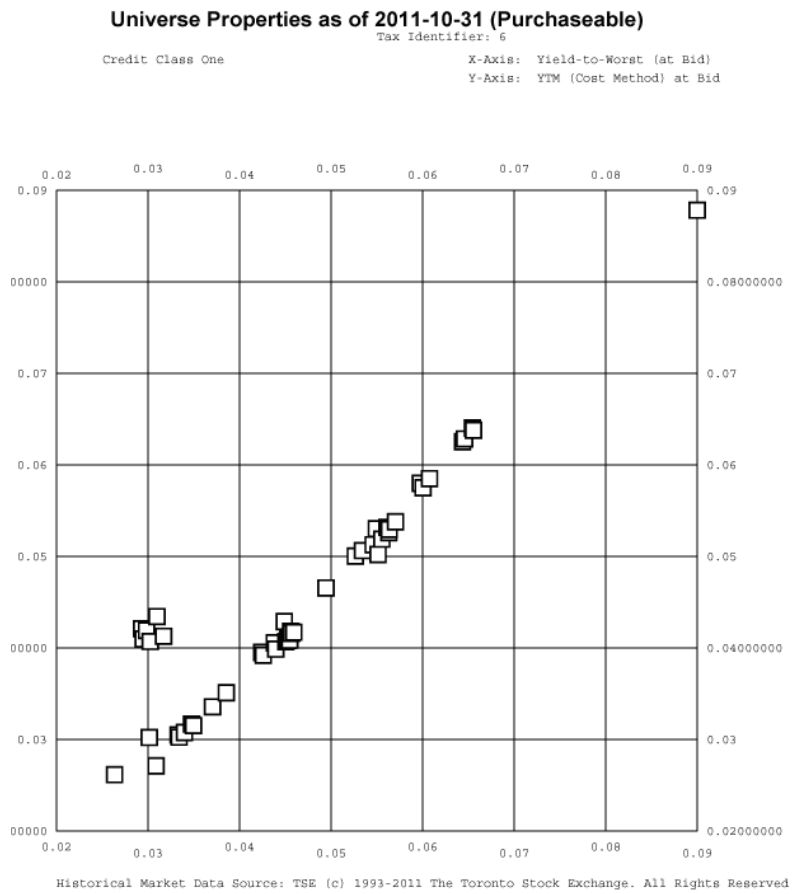
Y-4



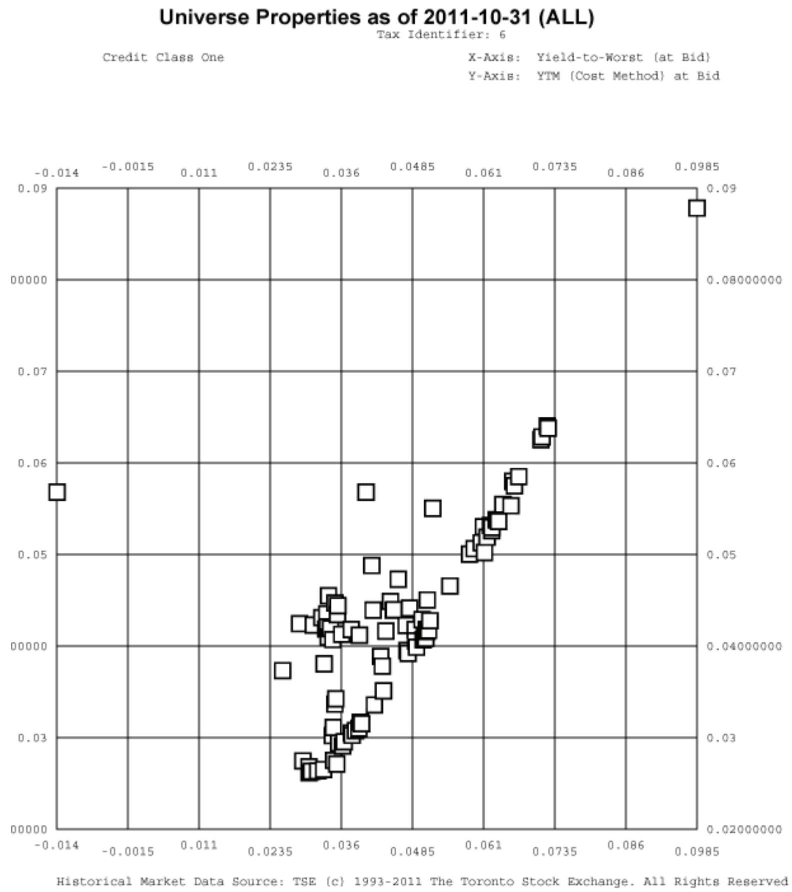
Y-5


Instrument : CIU.PR.A (Security A42705)
Tax Identifier: 6
X-Axis: Bid
Y-Axis: Pseudo-Modified Duration (Cost Me
23.18 23.43 23.68 23.93 24.18 24.43 24.68 24.93 25.18 25.43 25.68
22.5 21.25 20 18.75 17.5 16.25 15 13.75 12.5 11.25
Historical Market Data Source: TSE (c) 1993-2011 The Toronto Stock Exchange. All Rights Reserved

Y-6


Universe Properties as of 2011-10-31 (Purchaseable)
Tax Identifier: 6
Credit Class One
X-Axis: Yield-to-Worst (at Bid)
Y-Axis: YTM (Cost Method) at Bid
0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
0.09 00000 0.07 0.06 0.05 00000 0.03 00000
0.09 0.08000000 0.07 0.06 0.05 0.04000000 0.03 0.02000000
Historical Market Data Source: TSE (c) 1993-2011 The Toronto Stock Exchange. All Rights Reserved

Y-7



CurveYield

The concept of CurveYield was developed in order to address the shortcoming of CostYield with respect to high-coupon instruments having a market price highly influenced by the presence of a near-term call.

With this methodology, the cost of each option is determined by first calculating the present value of the cash flows that will be lost if the option is exercised at that point rather than using the “unconstrained price” that is used in CostYield. For example, if a perpetual issue has an annual coupon of $1.25 and the yield curve is a constant 4%, then the price of replacing the cash flows lost on option exercise will be $1.25/4%, or $31.25.

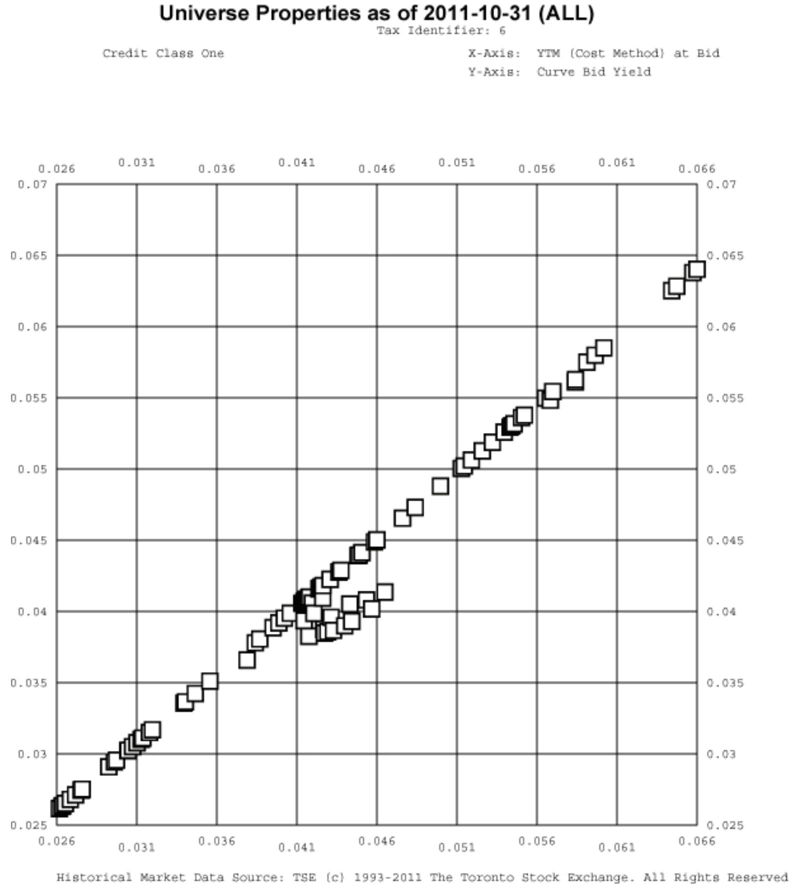
The price so determined is then compared with the Unconstrained Price used by the CostYield method and the greater of the two is used when valuing calls. This is because the options, by their nature, are assumed to reflect changes in the yield curve; the purpose of computing CurveYield is to address those cases in which a near-term call might be mis-priced because the market price of the instrument has taken account of that already.

As a result of this method of pricing the effects of option exercise, CurveYield will always be equal to or lesser than CostYield, as illustrated in Chart Y-8. By way of example, if we examine BAM.PR.P on 2011-10-31, we can compare the various calculated yields, as shown in Table Y-4. For this issue, there is only one option considered to be of consequence, at a price of $25.00 on 2014-9-30, considered to have an exercise probability of 68%. The CurveYield is considerably lower than the CostYield, since the Unconstrained Price on the exercise date of $28.53 is less than the Cash Flow Replacement Price of $34.10.

Table Y-4: Comparison of Various Yields Calculated for BAM.PR.P, 2011-10-31

Date	Future Value
Current Yield	6.47%
Yield to Worst	4.22%
PortfolioYield	4.70%
CostYield	5.17%
CurveYield	4.41%

Y-8



The major disadvantage of Curve Yield is that in order to calculate the Present Value of foregone cash flows, it is necessary to have a yield curve that will provide a discounting factor for each of these flows. Thus, the calculation of this variable is a recursive process, which adds to the complexity of the programming and the computation time.

In addition, given the similarity of CurveYield with CostYield, the problems that exist with respect to discontinuities in the relationship between price and yield continue to be present with CurveYield. It will be noted that in the case of BAM.PR.P, with only one option exercise opportunity incorporated in the cash flow schedule, this relationship is satisfyingingly smooth.

However, as may be seen in Charts Y-9 (Yield to Worst), Y-10 (CostYield) and Y-11 (Curve Yield) the latter two methods do a good job of smoothing transitions over a range of prices, which was the purpose behind their design.

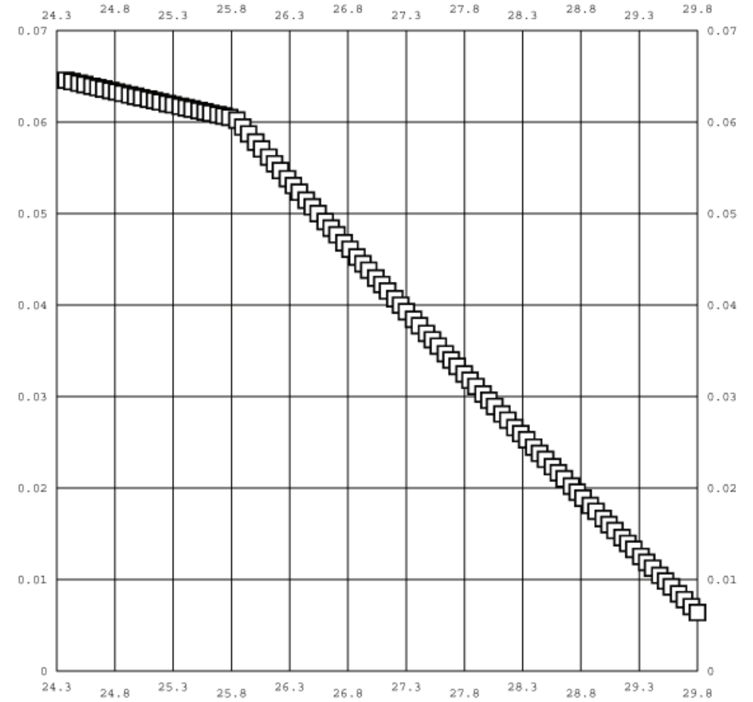
Y-9

Instrument : BAM.PR.P (Security A41225)

Tax Identifier: 6

X-Axis: Bid

Y-Axis: Yield-to-Worst (at Bid)



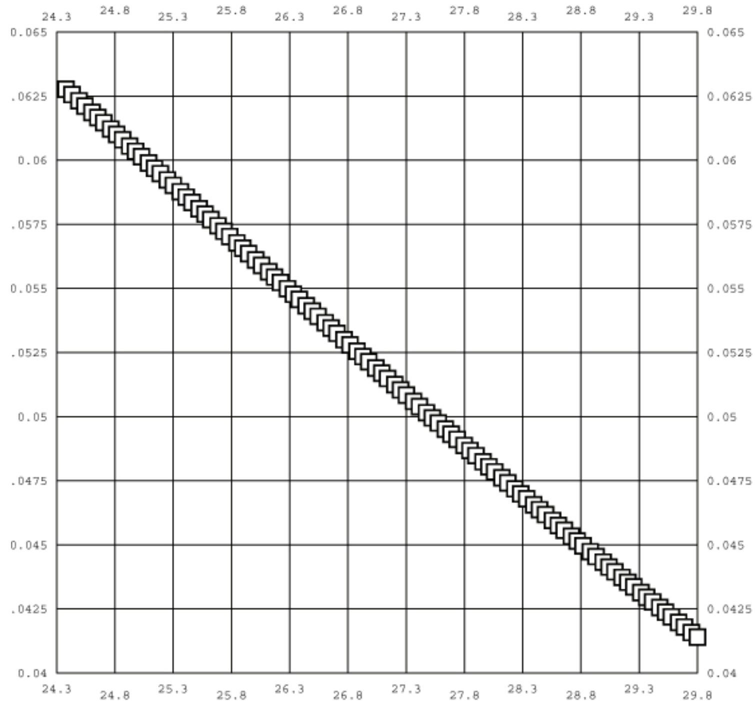
Y-10

Instrument : BAM.PR.P (Security A41225)

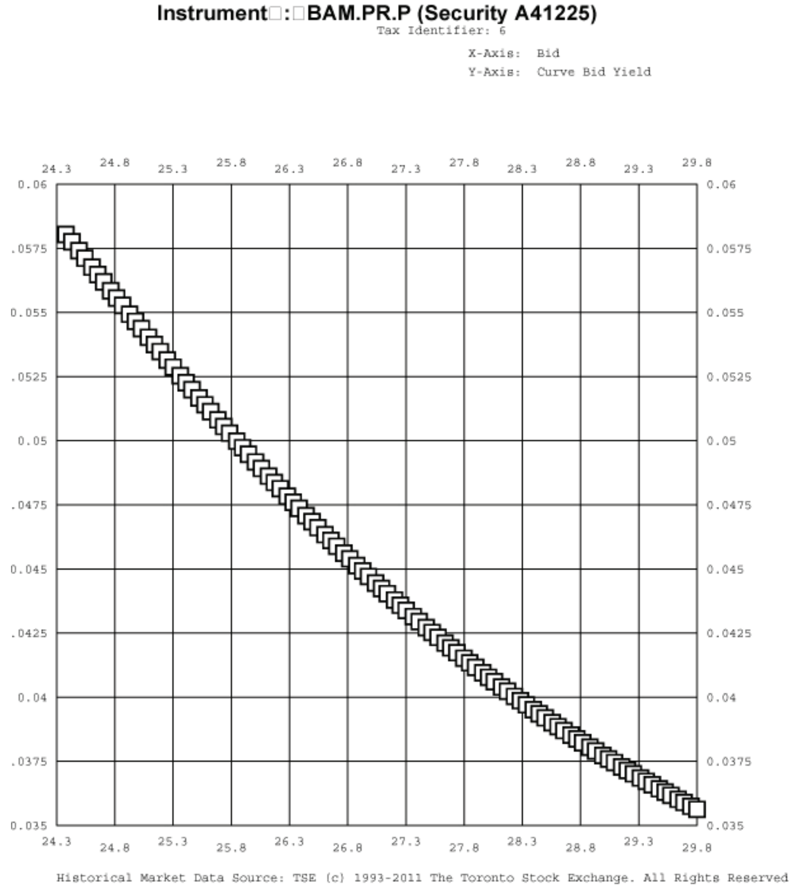
Tax Identifier: 6

X-Axis: Bid

Y-Axis: YTM (Cost Method) at Bid



Y-11



Yield to Issuer Best

As has been previously stated, the basic problem with Yield-to-Worst is that it assumes that the worst outcome for the holder, in terms of realized yield-to-call, is a certainty, but there are two problems with this assumption that have not yet been addressed:

- It might actually be worse for the holder, relative to other market opportunities, to realize a slightly higher yield, but for a longer term
- It assumes functional malevolence on the part of the issuer, who makes the decision as to whether any particular call opportunity should be utilized. In fact, the issuer is indifferent as to the effect on the holder; the issuer's objective is (generally speaking) to minimize its funding costs. These effects are usually identical, but this is not always the case.

Another way to think of this is to realize that we can make the YTW of any issue arbitrarily large and negative, simply by making the market price of the issue arbitrarily high. However, the issuer is not particularly concerned with the market price of the issues; the issuer is concerned with the redemption price and the cost of refinancing the issue when it is called. While the refinancing cost (before issue expenses) may normally be expected to be comparable to the market yield, this is not necessarily always the case.

Consider the case of ACO.PR.A in September, 2008.[5] This issue was[6] an OperatingRetractible, paying $1.4375 p.a. with embedded options as specified in Table Y-5.

Table Y-5: Embedded Options for ACO.PR.A

Option Type	Period Start	Period End	Strike Price
Call	2008-12-1	2009-11-30	26.00
Call	2009-12-1	2010-11-30	25.50
Call	2010-12-1	Infinite Date	25.00
Put	2011-12-1	Infinite Date	26.04*

**Price of 26.04 assumes that the $25 face value of the preferred is converted, via common shares, into cash at 96% of par. The actual formula is 95% of par (subject to a price floor), converted into cash at an unknown rate.*

5 Discussed at length at http://www.prefblog.com/?p=2993

6 The issue was redeemed on March 23, 2010. See http://www.prefblog.com/?p=9749

On 2008-9-12, the issue closed with a quote of 26.62-75, giving rise to the calculation for PortfolioYield shown in Table Y-6.

Table Y-6: PortfolioYield Calculation for ACO.PR.A, 2008-9-12

Date	Yield-to-Call	Restricted Yield	Probability
2008-12-31	-1.78%	-0.54%	40.1%
2009-12-31	2.35%	2.35%	1.0%
2011-11-30	3.68%	3.68%	58.9%

The YTW, according to the calculations of Table Y-6, is -1.78%, which assumes a call thirty days following the start of the option period.[7] However, if we look at the situation explicitly from the perspective of the issuer with the assistance of Table Y-5, we see that while the issue pays $1.4375 dividend p.a., the issuer can save $0.50 on the redemption price – thus, the net cost to the issuer of allowing the issue to remain outstanding for a year following the initial call date is only $0.9375, or 3.75% of par. If we further assume that the $26.00 redemption price would have to be refinanced, then the yield paid on the refinancing would have to be less than 0.9375 / 26 = 3.61% (the break-even value). Note that this does not account for the refinancing costs – which are about 3% of principal for preferred shares – but this approximation is acceptable since it is assumed that the refinancing costs will be incurred at some point anyway.

Depending upon market conditions – and market conditions in September 2008 were pretty grim – the cost of refinancing could exceed the break-even value; therefore, it is in the issuer's interest to let the issue remain outstanding past its first call date and therefore a rational investor might not choose to be overly concerned about the potential for an early call.

In order to reflect this reasoning on a systematic basis within HIMIPref™, I am planning the eventual introduction of Yield-to-Issuer-Best, or YTIB. The Net Present Value (NPV) of each call opportunity will be calculated using the yield curve derived for the entire preferred share universe (a similar calculation as is performed in the computation of CurveYield, but examining cash-flows up to and including the call, rather than the foregone cash-flows afterwards); the scenario with the lowest NPV will be selected as the "issuer best"; and the yield realized in this scenario will be the YTIB.

One can imagine an issuer examining the possibilities and thinking 'I can consider this issue as being two-, three- or four-year money. What should I consider it so that my cost is cheapest relative to what I will have to pay for a new issue of two-, three- or four-year money?'

However, programming, testing and possible implementation of this idea, and its usefulness in selecting issues that are likely to outperform their comparables, will have to await the next version of HIMIPref™.

Predictive Powers of Differing Yield and Duration Methodologies

An academic interest in the rationale and methodology of different yield calculations must never be permitted to mask the purpose of such computations: we are seeking to define a single well-defined attribute to be calculated for each member of a set of disparate preferred share issues that will serve to predict relative total return over time.

Two elements will be used in order to test the predictive powers of each type of calculation – yield and duration. It may be expected that in an environment of small overall market movements, yield will be the dominant factor, while the importance of duration will increase with size of these overall movements.

Yields and durations are measured on the last business day of each year commencing with 2000-12-29 and total return for each issue is calculated until the last business day of the following year (calculations commencing 2010-12-31 end on 2011-7-31). A regression of total return vs. either yield or duration is performed, with the data organized into three data sets (according to credit quality). The goodness of fit is measured using the statistical R-squared of the calculation.

Tables Y-7 and Y-8 show the results of correlation analysis for the various yield and duration measures calculated by HIMIPref™ over periods of one year; the performance of the BMO-CM "50" preferred share index is also reported to allow for a rough guess as to what the relative importance of yield and duration "should" be.

It will be noted that Table Y-7 shows that the ability of YTW to predict future performance was extremely poor as of 2006-12-29; this is because the market fell substantially in 2007 and many of the YTW scenarios (which had assumed a near-term call) were no longer applicable. The measure became more efficient the following year, commencing 2007-12-31, because the YTW scenarios no longer assumed near-term calls.

[7] At that time I was still assuming a mandatory thirty-day delay for all call exercises.

Table Y-7: Various Yield Measures vs. Subsequent Performance: R-Squared						
Year Commencing	**Current Yield**	**Yield to Worst**	**Portfolio Yield**	**Cost Yield**	**Curve Yield**	**Memo: Index Performance**
2000-12-29	0.13	0.09	0.32	0.32	0.32	+3.98%
2001-12-31	0.07	0.18	0.37	0.49	0.50	+4.39%
2002-12-31	0.17	0.25	0.33	0.21	0.34	+7.32%
2003-12-31	0.24	0.18	0.47	0.33	0.38	+6.02%
2004-12-31	0.15	0.07	0.25	0.12	0.12	+3.84%
2005-12-30	0.01	0.07	0.16	0.19	0.20	+4.27%
2006-12-29	0.47	0.07	0.57	0.51	0.52	-6.17%
2007-12-31	0.08	0.24	0.26	0.22	0.22	-16.42%
2008-12-31	0.79	0.66	0.66	0.67	0.67	+29.41%
2009-12-31	0.12	0.74	0.68	0.11	0.13	+10.11%
2010-12-31	0.59	0.07	0.21	0.10	0.12	+6.21%
The period commencing 2010-12-31 ends on 2011-7-31						

Table Y-8: Various Duration Measures vs. Subsequent Performance: R-Squared					
Year Commencing	**Mod Dur YTW**	**Pseudo-Mod Dur YTW**	**Pseudo-Mod Dur Port**	**Pseudo-Mod Dur Cost**	**Memo: Index Performance**
2000-12-29	0.09	0.10	0.15	0.08	+3.98%
2001-12-31	0.15	0.16	0.13	0.16	+4.39%
2002-12-31	0.45	0.48	0.46	0.25	+7.32%
2003-12-31	0.39	0.46	0.04	0.41	+6.02%
2004-12-31	0.03	0.12	0.36	0.05	+3.84%
2005-12-30	0.11	0.13	0.16	0.05	+4.27%
2006-12-29	0.18	0.30	0.38	0.10	-6.17%
2007-12-31	0.37	0.35	0.11	0.25	-16.42%
2008-12-31	0.21	0.19	0.16	0.22	+29.41%
2009-12-31	0.72	0.80	0.06	0.32	+10.11%
2010-12-31	0.37	0.31	0.02	0.18	+6.21%
The period commencing 2010-12-31 ends on 2011-7-31. Note that "Mod Dur YTW" is the modified duration of the YTW scenario, while "Pseudo-Mod Dur YTW" is calculated in accordance with the methodology presented in Box Y-1.					

Table Y-9 presents the results of similar calculations using three HIMIPref™ parameters:

- **Price Disparity:** the dollar value by which the present value of the instrument, calculated by discounting cash flows in accordance with the yield curve derived for the universe of instruments, differs from its market price.
- **Yield Disparity:** is the amount by which the yield curve would have to move up or down (in a parallel shift) in order for the Price Disparity to be zero (one might think of this as dividing the Price Disparity by the expected term, although the actual calculation is rather more complex)
- **Valuation:** is the valuation measure ultimately used by HIMIPref™ to evaluate the relative worth of each instrument in the universe, taking into account all the various calculations that attempt to relate one instrument to another.

It should be noted that regression is a rather suspicious measure to be using for quantitative analysis, since the beginning and ending points are pre-defined. If we choose between two instruments on the basis of some measure, and the instrument chosen immediately skyrockets but then falls back prior to the end-date, a regression calculation will score the choice as a poor one, despite the fact that in real-life we could well have changed our choice in the interim and been very pleased with ourselves.

When performing quantitative analysis, as weary readers of this essay will have come to appreciate, it is important to understand precisely what it is you are measuring and whether that measure is actually well suited to your ultimate purpose. The HIMIPref™ valuation measures examined in Table Y-9 were not calculated in an effort to maximize R-Squared over any constant period of time; they were optimized in order that successive swaps between issues would lead to a good result for the portfolio, regardless of whether any given position was held for two days or two years. Additionally, the correlation looks at all issues, which do not necessarily behave in the same way as the subset of issues which are actually purchased and considered for purchase, which will represent extreme values of the valuation measures.

There will be some correlation between the worth of the valuation measures as a portfolio management tool and the goodness-of-fit when these are applied over the complete universe of possibilities over constant, pre-set time periods, but the correlation is incidental to the actual process of valuation measure development. Regression analysis is a useful development tool and serves an excellent purpose in identifying possible patterns for detailed investigation, but only bank employees would use it as a serious investigative measure.

Table Y-9: Various HIMIPref™ Measures vs. Subsequent Performance: R-Squared

Year Commencing	Price Disparity	Yield Disparity	Valuation
2000-12-29	0.10	0.10	0.10
2001-12-31	0.16	0.35	0.26
2002-12-31	0.18	0.22	0.59
2003-12-31	0.84	0.79	0.84
2004-12-31	0.21	0.31	0.39
2005-12-30	0.02	0.06	0.06
2006-12-29	0.49	0.47	0.54
2007-12-31	0.15	0.26	0.18
2008-12-31	0.58	0.70	0.72
2009-12-31	0.07	0.06	0.09
2010-12-31	0.58	0.08	0.13

The period commencing 2010-12-31 ends on 2011-7-31

Relative Importance of Yield Measures in Valuation

HIMIPref™ combines all the calculated yield measures into one grand valuation parameter, which is used as the basis for scaling other parameters (the 'dynamic' parameters, such as Disparity, which attempt to predict relative price changes of the instruments beyond simply 'earning the coupon', and are designed to sum to zero).

The contributions of the yield measures are shown in Table Y-10. These weightings were determined through repeated simulations – adjusting each contribution prior to each simulation and choosing the combination of parameters that gave the best portfolio management results.

Table Y-10: Importance of Various Yield Measures to HIMIPref™ Yield Valuation (Taxable Portfolio, Portfolio Method)

Yield Measure	Contribution to "Total Yield"
Current Yield	0.00%
Yield to Worst	37.25%
Portfolio Yield	1.64%
Cost Yield	59.64%
CurveYield	1.62%

Total does not equal 100% due to rounding.

Conclusion

Any yield measure will depend upon various assumptions, and any of these assumptions may become invalid in certain situations.

Current Yield assigns a 100% chance to the instrument's being perpetual, even if the prospectus does not allow the issue to exist to perpetuity.

Yield to Worst assigns a 100% chance to the scenario that results in the worst realized yield for the investor, even if this is not the most favourable scenario for the issuer. This can result in sharp changes in measured yield over relatively short price intervals

Portfolio Yield is computationally expensive; in order to get results after a reasonable time, it is necessary to impose a threshold requirement on the chances for any possible option exercise, which results in discontinuities in the price-yield curve. It also relies on Black-Scholes option theory; the current implementation is by market price, when a better implementation would base the calculation on changes in market yield. This emphasis on price also leads to logical errors when the market price has been influenced by market expectation of call exercise.

Cost Yield suffers many of the same flaws as Portfolio Yield, but does succeed in reducing the size of discontinuities in the price-yield curve.

Curve Yield is again afflicted with the computational problems in deriving a call schedule with probabilities that apply to Portfolio Yield and Cost Yield. It attempts to address the price directionality inherent in market expectations of a call, but does so only as an adjustment, not in a logically consistent manner. It does not appear to be a significant improvement over Cost Yield.

In short – yield is complex! When calculating and comparing yields, investors should ensure that they review the assumptions inherent in their calculation and deprecate the results of their calculations to the extent that the assumptions are invalid.

For readers' interest, evaluations of Straight Perpetuals are presented in Tables Y-11, Y-12, Y-13 and Y-14. The "Bid Valuation – Sum of Yield Components" is determined according to the weight of each element. The weightings are different than those presented in Table Y-10, as these weightings, presented in Table Y-15, are derived for non-taxable accounts, which have not been studied as intensively as taxable accounts.

Table Y-15: Importance of Various Yield Measures to HIMIPref™ Yield Valuation (Non-Taxable Portfolio, Issue Method)

Yield Measure	Contribution to "Total Yield"
Current Yield	0.00%
Yield to Worst	50.72%
Portfolio Yield	38.69%
Cost Yield	8.89%
CurveYield	1.67%
Total does not equal 100% due to rounding.	

Table Y-11

HIMI Index - Perpetual (Premium)
HIMI Account Number:J0000007
Evaluation Date : 2011-11-11
Evaluation Timing : Trade Date
Data Source : Transaction Files

Ticker	Bid	Ask	Current Yield Bid	YTM (Port Method) at Bid	YTM (Cost Method) at Bid	Yield-to-Worst (at Bid)	Curve Bid Yield	Bid Valuation - sum of Yield Components
CM.PR.D	25.33	25.38	5.68 %	3.81 %	5.44 %	3.23 %	5.43 %	3.69
CU.PR.A	25.40	25.70	5.71 %	3.31 %	5.46 %	-5.23 %	5.21 %	-0.80
CU.PR.B	25.55	25.60	5.87 %	3.01 %	5.64 %	-12.02 %	5.26 %	-4.34
ENB.PR.A	25.67	25.75	5.36 %	1.80 %	5.17 %	-28.17 %	5.06 %	-13.05
FTS.PR.F	25.22	25.40	4.86 %	4.99 %	4.59 %	4.83 %	4.59 %	4.86
IGM.PR.B	26.17	26.28	5.64 %	5.54 %	5.23 %	5.17 %	5.23 %	5.32
POW.PR.C	25.23	25.35	5.75 %	4.03 %	5.57 %	2.53 %	5.57 %	3.43
PWF.PR.G	25.25	25.37	5.84 %	3.74 %	5.69 %	-4.04 %	5.59 %	-0.00
PWF.PR.H	25.25	25.36	5.69 %	3.83 %	5.49 %	0.63 %	5.47 %	2.38
PWF.PR.I	25.51	25.56	5.88 %	3.36 %	5.70 %	-4.34 %	5.59 %	-0.30
PWF.PR.O	26.27	26.33	5.52 %	5.36 %	5.10 %	4.99 %	5.10 %	5.14
TCA.PR.X	52.00	52.48	5.38 %	4.40 %	5.04 %	3.54 %	5.04 %	4.03
TCA.PR.Y	52.60	52.62	5.32 %	4.18 %	4.91 %	3.33 %	4.91 %	3.82
	Cash							
	Dividends Due							
	Income Tax Due							
	Capital Gains Tax Due							

Table Y-12

HIMI Index - Perpetual (Discount)
HIMI Account Number:J0000009
Evaluation Date : 2011-11-11
Evaluation Timing : Trade Date
Data Source : Transaction Files

Ticker	Bid	Ask	Current Yield Bid	YTM (Port Method) at Bid	YTM (Cost Method) at Bid	Yield-to-Worst (at Bid)	Curve Bid Yield	Bid Valuation - sum of Yield Components
BAM.PR.M	23.46	23.51	5.06 %	5.89 %	4.99 %	5.11 %	4.99 %	5.40
BAM.PR.N	23.50	23.56	5.05 %	5.70 %	5.00 %	5.10 %	5.00 %	5.32
CIU.PR.A	24.95	24.99	4.61 %	5.18 %	4.31 %	4.59 %	4.31 %	4.79
CM.PR.E	25.03	25.05	5.59 %	4.76 %	5.36 %	5.63 %	5.36 %	5.26
CM.PR.G	24.92	24.97	5.42 %	5.57 %	5.19 %	5.45 %	5.19 %	5.47
ELF.PR.F	22.61	22.80	5.86 %	6.36 %	5.85 %	5.92 %	5.85 %	6.08
ELF.PR.G	20.86	20.94	5.69 %	5.76 %	5.76 %	5.76 %	5.76 %	5.76
POW.PR.A	25.10	25.14	5.58 %	4.09 %	5.43 %	5.63 %	5.43 %	5.01
POW.PR.B	25.01	25.09	5.35 %	4.16 %	5.15 %	5.39 %	5.15 %	4.89
POW.PR.D	24.26	24.70	5.15 %	6.06 %	5.00 %	5.18 %	5.00 %	5.50
PWF.PR.E	25.10	25.20	5.48 %	5.40 %	5.25 %	5.31 %	5.25 %	5.34
PWF.PR.F	24.75	25.00	5.30 %	4.63 %	5.14 %	5.33 %	5.14 %	5.04
PWF.PR.K	24.28	24.40	5.10 %	5.89 %	4.93 %	5.11 %	4.93 %	5.39
PWF.PR.L	24.65	24.80	5.17 %	5.92 %	4.99 %	5.19 %	4.99 %	5.45
RY.PR.W	24.94	24.99	4.91 %	4.92 %	4.66 %	4.91 %	4.66 %	4.89
W.PR.H	25.20	25.28	5.46 %	5.34 %	5.24 %	5.18 %	5.24 %	5.25
W.PR.J	25.10	25.32	5.58 %	4.09 %	5.43 %	5.63 %	5.36 %	5.01
Cash								
Dividends Due								
Income Tax Due								
Capital Gains Tax Due								

Table Y-13

HIMI Index - Perpetual (Discount)
HIMI Account Number:J0000009
Evaluation Date : 2011-11-11
Evaluation Timing : Trade Date
Data Source : Transaction Files

Ticker	Bid	Ask	Pseudo-Modified Duration (Port Method)	Pseudo-Modified Duration (Cost Method)	PseudoModifiedDuration - Worst	Modified Duration - YTW
BAM.PR.M	23.46	23.51	-9.96	13.41	18.66	15.23
BAM.PR.N	23.50	23.56	-32.73	11.43	18.98	15.24
CIU.PR.A	24.95	24.99	4.41	13.82	7.53	16.23
CM.PR.E	25.03	25.05	1.54	18.94	1.84	14.45
CM.PR.G	24.92	24.97	2.34	12.14	6.56	14.73
ELF.PR.F	22.61	22.80	153.06	14.06	16.70	13.98
ELF.PR.G	20.86	20.94	17.19	17.19	17.19	14.26
POW.PR.A	25.10	25.14	1.38	19.27	0.17	1.18
POW.PR.B	25.01	25.09	1.47	21.51	0.25	14.79
POW.PR.D	24.26	24.70	8.53	13.00	20.42	15.12
PWF.PR.E	25.10	25.20	2.58	11.41	1.80	1.17
PWF.PR.F	24.75	25.00	1.76	14.86	18.31	14.93
PWF.PR.K	24.28	24.40	7.72	13.18	20.81	15.27
PWF.PR.L	24.65	24.80	5.71	12.92	20.58	15.13
RY.PR.W	24.94	24.99	2.59	13.79	4.38	15.67
W.PR.H	25.20	25.28	2.47	10.32	1.63	1.13
W.PR.J	25.10	25.32	1.38	19.27	0.17	1.18
Cash						
Dividends Due						
Income Tax Due						
Capital Gains Tax Due						

Table Y-14

HIMI Index - Perpetual (Premium)
HIMI Account Number:J0000007
Evaluation Date : 2011-11-11
Evaluation Timing : Trade Date
Data Source : Transaction Files

Ticker	Bid	Ask	Pseudo-Modified Duration (Port Method)	Pseudo-Modified Duration (Cost Method)	PseudoModifiedDuration - Worst	Modified Duration - YTW
CM.PR.D	25.33	25.38	1.25	17.30	0.15	0.46
CU.PR.A	25.40	25.70	1.17	14.95	0.10	0.08
CU.PR.B	25.55	25.60	1.08	17.73	0.09	0.08
ENB.PR.A	25.67	25.75	0.85	14.53	0.10	0.09
FTS.PR.F	25.22	25.40	2.36	9.76	3.56	2.83
IGM.PR.B	26.17	26.28	4.98	10.21	5.75	5.83
POW.PR.C	25.23	25.35	1.41	19.87	0.20	0.15
PWF.PR.G	25.25	25.37	1.19	41.06	0.09	0.08
PWF.PR.H	25.25	25.36	1.39	18.71	0.18	0.16
PWF.PR.I	25.51	25.56	1.08	19.74	0.10	0.08
PWF.PR.O	26.27	26.33	4.66	10.28	5.67	5.74
TCA.PR.X	52.00	52.48	2.64	13.82	1.81	1.82
TCA.PR.Y	52.60	52.62	2.72	9.12	2.15	2.17
	Cash					
	Dividends Due					
	Income Tax Due					
	Capital Gains Tax Due					