Annuities[1]

Pensions and annuities have been in the news recently,[2] with there being a great deal of controversy about the value of annuities in retirement planning at the present time. Much of the recent attention has been triggered by the bankruptcy of Nortel; the pension plan was under-funded and, in accordance with current bankruptcy law, the plan assets were required to be rolled into annuities for each beneficiary.

However, after considerable outcry from the affected pensioners, the Ontario government has proposed legislation that would allow them to roll their assets instead into a Life Income Fund account,[3] although the alternative plan of allowing the plan to continue its existence was rejected.[4]

The pensioners were outraged that the only option formerly available to beneficiaries of wound-up pension plans was the purchase of a life annuity on their behalf:[5] *"Wind-up-by-annuity adds unnecessary expense and uncertainty for everyone" stated Don Sproule, National Chair of the NRPC, "There is no worse time to buy annuities than in the current economic conditions". He went on to say "On top of that, the $2.5B Nortel pension funds may cause a systemic failure of the Canadian annuity market, which can only cope with around $500M per year. There is no way we can expect robust competition in the conversion of our pension assets to annuities. Prices will go through the roof."*

The assertion that such a large amount could cause the systemic failure of the Canadian annuity market sounds a little surprising to me – surely funds could be invested abroad and swapped into Canadian dollars – but Mr. Sproule has doubtless investigated the issue more thoroughly than I have. I am more interested in his assertion that September 2010 was the worst possible time to buy annuities, particularly in light of the recent publication of a book[6] touting annuities as one of the three "product silos" for asset allocation in retirement.

First, borrowing liberally from the April, 2010, edition of this newsletter, I will review annuities as an investment and as insurance (it is important to keep both objectives in mind!). I will then discuss the calculation of the Probability of Ruin (which is the probability your portfolio value will go underground before you do) in terms of a third-party financial calculator and my own version, which includes annuities as an investable asset class and then provide conclusions and recommendations regarding financial planning for retirement.

Types of Annuities

An annuity is[7] a contract whereby *in exchange for a single payment, a life insurance company provides the annuitant with regular income payments, generally for the rest of the annuitant's life*.

The benefits of an annuity are best explained by the concept of a tontine annuity: such a scheme has a rich and not altogether sterling history.[8] *A tontine is an investment scheme through which shareholders derive some form of profit or benefit while they are living, but the value of each share devolves to the other participants and not the shareholder's heirs on the death of each shareholder. The tontine is usually brought to an end through a dissolution and distribution of assets to the living shareholders when the number of shareholders reaches an agreed small number.*

Annuities are no longer provided by tontine groups – although I'm sure that there are still a few extant as private contracts – but by insurance companies. Those seeking to purchase an annuity need no longer join a large group of people with similar characteristics and investment goals; instead, it is the insurance company that acts as counterparty and for each contract forms a 'virtual tontine' providing similar benefits, while attempting to make a profit.

There are a great number of insurance products offered besides such plain-vanilla annuities, but it seems fair to say that these are simply various combinations of simpler elements. An annuity with a guaranteed payment period, for example, can be replicated with a straight annuity and life insurance. As David O'Leary of Morningstar observed[9] when Manulife introduced its "Income Plus" product[10]: *It simply combines some of the benefits of an annuity with those of a segregated fund*.

1 This essay incorporates material from the April, 2010, edition of this newsletter and from my article '*The Annuity Decision*', available on-line at http://www.himivest.com/media/moneysaver_1103.pdf

2 E.g. Julian Beltrame, *Few guarantees in Ignatieff's 'secure' pension plan*, The Record.com, 2011-4-2, available on-line at http://www.therecord.com/print/article/510935 (accessed 2011-4-9)

3 Ontario Ministry of Finance, *Frequently Asked Questions for Nortel Pensioners*, available on-line at http://www.fin.gov.on.ca/en/consultations/pension/nortel-faqs.html (accessed 2011-4-9)

4 Janet McFarland, *Ontario says Nortel workers can opt for private pension management*, Globe and Mail, 2011-1-3, available on-line at http://www.theglobeandmail.com/globe-investor/ontario-says-nortel-workers-can-opt-for-private-pension-management/article1856063/ (accessed 2011-4-9)

5 Nortel Retirees and former employees Protection Canada, *Don't wind up our pension plans!*, Media Release, 2010-8-31, available on-line at http://nortelpensioners.ca/index.php?option=com_content&task=blogcategory&id=113&Itemid=163 (accessed 2011-4-9)

6 Moshe A. Milevsky and Alexandra C. Macqueen, *Pensionize Your Nest Egg*, 2010, ISBN 9780-0-470-68099-5

7 CANNEX Financial Exchanges Ltd, *Products: Annuities*, available on-line at http://www.cannex.com/canada/english/products_antc.htm (accessed 2010-4-6)

8 Kent McKeever, *A Short History of Tontines*, available on-line at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1340062 (accessed 2010-4-6)

9 David O'Leary, *Our thoughts on Manulife's new guaranteed minimum withdrawal product*, Morningstar, 2006-11-15, available on-line at http://www.morningstar.ca/globalhome/industry/news.asp?articleid=ArticleID1114200613231 (accessed 2011-4-9)

10 Manulife, *Income Plus*, available on-line at http://manulifegifselect.ca/incomeplus/introduction_video

Thus, the elements of the plain vanilla annuity discussed in this essay are:

Prescribed: Prescribed annuities are generally preferred by investors and advisors as they are taxed on a preferential basis,[11] but this preference is accompanied by the requirement that they be purchased with non-registered funds, among other conditions[12]. The taxable portion of a prescribed annuity is:

Taxable Income = Annual payment – Single premium/Life Expectancy

So, for example, if a client pays $100,000 for an annuity paying $8,460 annually and has a life expectancy of 13.76 years, the taxable income is calculated as

Annual Taxable Income = 8,460 – 100,000/13.76 = $1,192

The remaining portion of the annual payments, 8,460 – 1,192 = 7,268, is considered (quite rightly!) to be return of capital and is not taxed – even if the annuitant should live longer than expected and hence have more capital returned than he originally deposited. A nice little bonus with this treatment is that the government uses the 1971 mortality tables[13] for this calculation; the slightly shorter life expectancy increases the return-of-capital amount to above what might be expected today, decreasing the taxable amount.

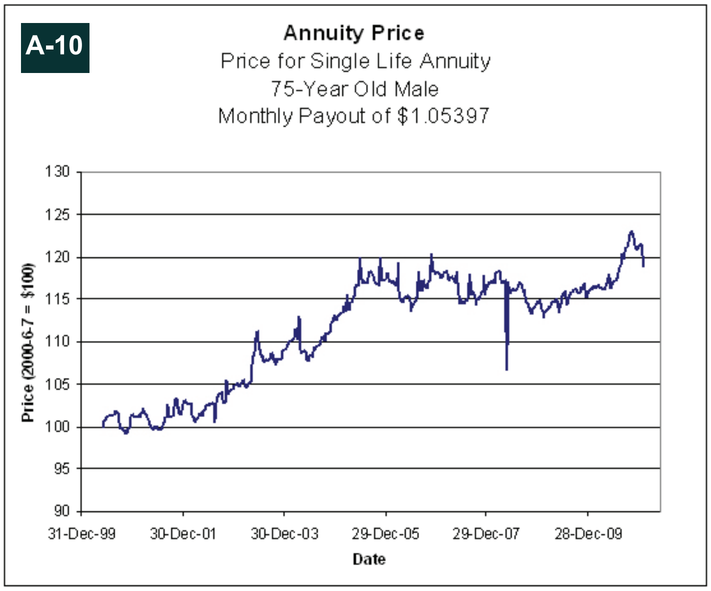
Single Life: A single life annuity contract provides payments to a single annuitant; when the annuitant dies, the contract is completed. The alternative is "joint and survivor life", which is purchased with two initial beneficiaries and continues until both have died.[14]

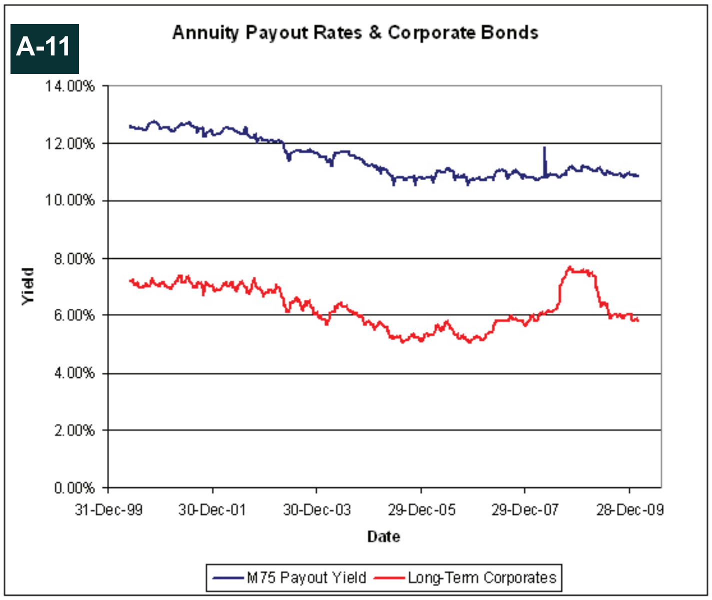
No Guarantee: The plain vanilla annuities discussed in this essay have no guarantee. Guaranteed annuities are those which will pay the agreed income for a minimum length of time; if the annuitant should die before the guarantee period expires, the annuity income will be paid to his beneficiaries until the period expires.[15] It should be noted that such a guarantee should be considered as life insurance – and, what's more, very expensive life insurance. Since insurers and their salesmen will generally quote annuities with a ten-year guarantee unless explicitly told not to, purchasers should be very careful to understand the difference in price and the cost of this feature. For example, given current rates[16] a seventy-five-year old male can expect monthly income of about $880 monthly from annuity principal of $100,000 with no guarantee; the rate is about $750 with a ten-year guarantee. To put it another way, monthly income of $880 costs $100,000 with no guarantee, or $117,000 with a guarantee. Does the investor really need to spend $17,000 on life insurance? Is that really the best solution and the best deal available? Retirees and their beneficiaries should consider the question carefully.

Pricing of Annuities

As discussed in the April, 2010, edition of this newsletter, the best determinant of annuity rates is the long term corporate rate. Pricing data is available[17] only to 2011-2-2, but this is sufficient to allow us to construct a graph of annuity prices in Chart A-10 and lo and behold! It appears that the Nortel retirees were correct and that the fall of 2010 was indeed a very poor time to be purchasing an annuity, with the peak price of $123.02 being reached on November 10.

At first glance, this appears to put the lie to my assertion that long term corporate rates are of primary importance in pricing annuities, since Chart A-11 shows long term corporate yields were lower at other times, but there appear to be other things going on.





11 Standard Life, *Taxation of Non-Registered Enriched (or Impaired) Annuities*, April 2007, available on-line at http://www.advisors.standardlife.ca/en/pdf/reference/taxing/6139.pdf (accessed 2010-3-26)

12 See Manulife, *Taxation of Non-Registered Prescribed Annuity Contracts*, October, 2009, available on-line at https://hermes.manulife.com/CANADA/repsrcfm-dir.nsf/Public/TaxTopicTaxationofNonRegisteredPrescribedAnnuityContracts/$File/tepg_taxtopic_TxPrAn.pdf

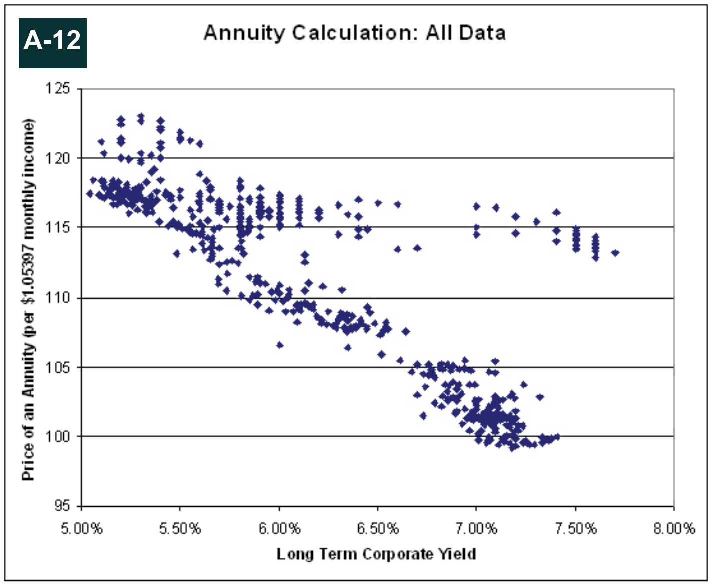
13 Standard Life, supra

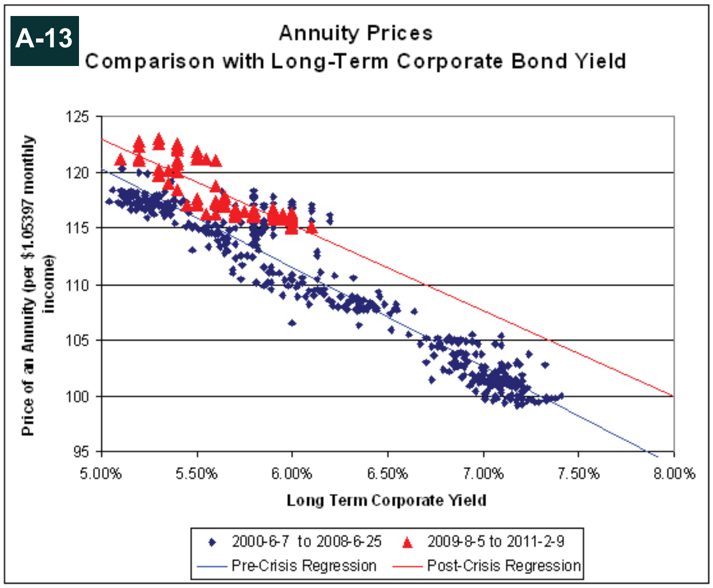
14 Manulife, *What Types of Annuities are Available?*, available on-line at http://www.manulife.ca/canada/investments.nsf/Public/guaranteed_annuities_types (accessed 2010-4-7)

15 Organization for Economic Co-operation and Development, *Glossary of Statistical Terms: Guaranteed Annuity*, available on-line at http://stats.oecd.org/glossary/detail.asp?ID=5242 (accessed 2010-4-7)

16 See http://www.canadianbusiness.com/my_money/rates/index.jsp?ref=ln (accessed 2011-4-9)

17 The Individual Finance and Insurance Decisions Centre, *Payout Annuity Index*, available on-line at http://www.ifid.ca/payout.htm (accessed 2011-4-9)





Readers examining the aggregated data relating long term corporate yields to annuity payout rates might well be inclined to be suspicious of my assertion that there is a strong relationship: but it must be remembered that the Panic of 2007 caused a lot of long-standing relationships to decouple. After examining the raw data, it becomes clear that the crisis period, from the begining of July 2008 to the end of July 2009, constitutes a special case. These data may be confidently discarded and the remaining data disaggregated into 'Pre-Crisis' and 'Post-Crisis' datasets.

Chart A-13 and the results of the regression illustrated there allows us to draw two conclusions of great interest:

- The insurers appear to have recently increased the price of annuities from their pre-crisis levels. This may be due to increased recognition of risk, or it may be due to a more basic desire to maximize profits.
- The slope of the regression line is -8.9 pre-crisis and -7.7 post crisis. These numbers serve as empirically derived estimates of the Modified Duration[18] of annuity prices (for an annuity as defined by the data: single-life, male, no guarantee), which is very important for hedging purposes.

Hedging can be quite important: if you seek to hold a bond portfolio that will vary in price in close to the same manner as an annuity, you should ensure that Modified Durations are matched. The Modified Duration of the BMO Long Corporate Bond Index ETF (ZLC) is 11.76 years[19] while the BMO Mid Corporate Bond Index ETF (ZCM) has a modified duration of 5.88 years,[20] suggesting that a portfolio comprised of about 50% Long ETF/50% Short ETF will react to interest rate changes in much the same way as the annuity of interest. Some investors may seek a more precise duration match by increasing the weight of the mid-term constituent, but it is not clear to me that the data will support such precision.

In fact, one investor has suggested that a deliberate overweighting of the shorter-term component: 'If interest rates go down, my funds will go up in price less than the annuity will and I shall defer purchase. If interest rates go up, my funds will go down in price less than the annuity does and I shall bring forward purchase'[21]. Note, however, that this strategy involves a certain amount of interest rate risk: what if yields go down and stay down? Do you defer purchase forever?

Annuities as an Investment

For the ensuing computations, I will use the Social Security Life Table (SSLT),[22] as of 2005, published by the US government; another potential source of mortality data is available from Standard Life.[23]

According to the SSLT a seventy-five year old man has a life expectancy of 10.26 years, and according to the Cannex data[24], an investment of $100,000 would have provided a life annuity of $890 monthly. If we calculate the Internal Rate of Return based on these data we arrive at a figure of 1.79% – which is a pretty poor return on investment.

Chart A6 in the section "Rates of Return", below, shows how the realized yields increase with survival time, while Table A6 compares annuities with long-term corporate bonds in terms of time to break-even.

18 For more discussion of Modified Duration, see my article at http://www.himivest.com/media/moneysaver_0705.pdf

19 See http://www.etfs.bmo.com/bmo-etfs/holdings?fundId=75747 (accessed 2011-4-9)

20 See http://www.etfs.bmo.com/bmo-etfs/holdings?fundId=75744 (accessed 2011-4-9)

21 See Financial Webring Forum, "*Duration Matching with an Annuity*" on-line at http://www.financialwebring.org/forum/viewtopic.php?f=30&t=112268&start=67 (accessed 2011-4-10)

22 Available on-line at http://www.ssa.gov/OACT/STATS/table4c6.html

23 Available on-line at http://www.advisors.standardlife.ca/en/pdf/reference/4984.pdf

24 See http://www.canadianbusiness.com/my_money/rates/annuity_single_life_male_no_guarantee/index.jsp (accessed 2011-4-9)

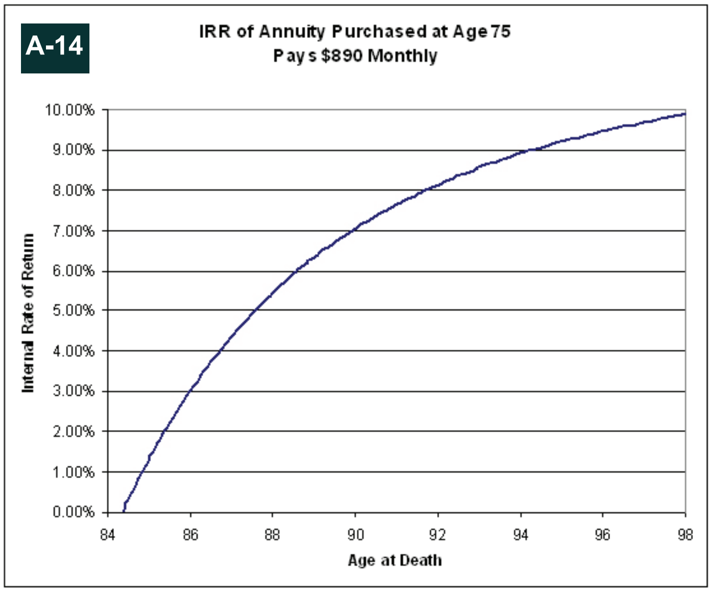
Annuities as Insurance

Despite the deprecation of annuities as an investment, above, it should not be forgotten that annuities represent excellent insurance.

Chart A-14 shows the realized yield of an annuity paying $890 monthly, purchased when the annuitant is 75 years old. If he should live for 19-odd years following his purchase, his realized yield will be 9.00% – far better than bonds and unmatchable by other investments offering the same degree of safety.

Those who find that funding their retirement entails encroaching upon their capital should very carefully consider the addition of annuities to their portfolio; ideally, I suggest, every dollar of expected capital depletion should be depleted via annuitization rather than simply being spent directly.

That being said, an even better form of longevity insurance for an individual is a deferred annuity – or would be, if any were available in Canada! Deferred annuities will be discussed shortly..



Rates of Return

In the Base Case scenario for the spreadsheet to be introduced shortly, I have assumed that the expected rate of return for bonds is 5.5%, with a 5.5% coupon – this is the approximate yield of the DEX Long-Term All Corporate Bond Index[25] while the yield-to-maturity on ZLC, the BMO Long Corporate Bond Index ETF, is currently 5.43% gross of a MER capped at 0.30%, for an expected net yield of 5.13%

Since an annuity can be thought of as a fixed-income substitute with special characteristics – a higher cash yield for life, but no return of principal – it is of interest to see how long investors of various ages must live to exceed the long-term bond rate of return. These figures are shown in Table A6; the monthly payments are from Cannex[26] and the life-expectancies are US government figures from 2006.[27]

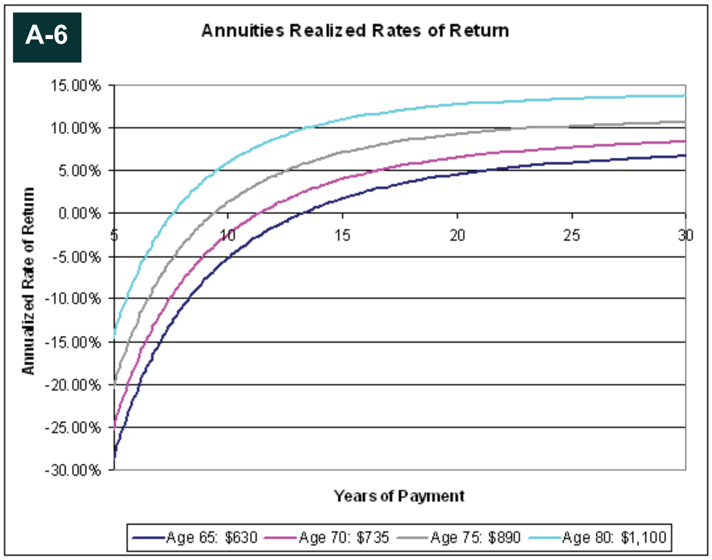
Another method of viewing the data is given by Chart A6, which shows the realized yield given a monthly rate and the term for which the annuitant reaps the benefit of his investment. The first five years are not shown – since the rates are obtained from the "No Guarantee" Cannex table, there is a huge loss realized if the annuitant dies during this period.

[25] PC Bond Analytics, *DEX Long Term Bond Indices, Closing Figures for 04/05/2011*, available on-line at http://www.canadianbondindices.com/ltbi.asp (accessed 2011-4-6)

[26] Determined 2011-4-6 from Canadian Business, *Annuity Rates – Single Life Male No Guarantee*, data provided by Cannex, available on-line at http://www.canadianbusiness.com/my_money/rates/annuity_single_life_male_no_guarantee/index.jsp (accessed 2011-4-6)

[27] Social Security Online, *Period Life Table*, 2006, available on-line at http://www.ssa.gov/OACT/STATS/table4c6.html (accessed 2011-4-6)

Table A6: Time for an Annuity to Break-Even with Bonds			
Age at Purchase	**Monthly Payment per $100,000 invested**	**Years to 5.50% realized yield**	**Life Expectancy**
65	$630	23.1	17.0
70	$735	17.5	13.6
75	$890	13.0	10.5
80	$1,100	9.7	7.8



Deferred Life Annuities

Deferred Life Annuities (DLAs) are available in the US, but do not appear to be common in Canada, at least not in their pure form. A DLA allows a purchaser to pay a lump-sum to the insurer in exchange for a life annuity that begins after a deferral period, rather than immediately as with the more normal annuities discussed in this essay.

Several (perhaps all) insurers offer what they refer to as DLAs, for instance, Standard Life's description[28] reads:

- *You purchase a deferred annuity policy (term certain to age 90) with the option of converting to a RRIF at the earlier of age 71 and within 7 years of the annuity purchase date; an interest rate and term are selected for the RRIF*
- *The single premium accumulates at the annuity policy's interest rate*
- *If you convert to a RRIF, the accumulated value of the policy is transferred to the RRIF while maintaining the RRIF interest rate for the balance of the term selected*
- *If you choose not to convert to a RRIF, annuity payments automatically start at the annuity policy's commencement date*

Naturally a lot will depend on the pricing, but this description does not seem as much like a formal DLA as it does a package of products designed to seduce the unwary into thinking that they are buying a DLA.

I say this because there does not appear to be any mechanism whereby an investor can capture the mortality credits that should be applicable.

To illustrate a "proper" DLA, let us consider a sixty year old male preparing for retirement and seeking insurance against the probability of him exceeding his life-expectancy of about[29] twenty years. For every 100,000 males born, 85,026 survive to age 60, according to the standard tables, so we will assume that 85,026 sixty-year-males purchase a DLA that commences payments at age 80; there should be 47,073 survivors.

[28] Standard Life, *Conventional Annuities: Deferred Annuity (Prepaid RRIF)*, available on-line at http://www.standardlife.ca/en/individual/investments/annuities/annuity.html#def (accessed 2011-4-8)

[29] Social Security Online, *Period Life Table*, 2006, available on-line at http://www.ssa.gov/OACT/STATS/table4c6.html (accessed 2011-4-6)

Calculating the pricing of such an annuity might erroneously be thought of as simply a process of discounting the price of an eighty-year-old's annuity for the twenty year deferral period, but that is not a DLA: that is merely a package of products designed to resemble a DLA. A true DLA will have mortality credits, so that the lucky 47,073 survivors will have not only their own funds invested in the annuity, but also the funds of those who did not survive the passage of time, which will essentially double their money.

The major obstacle to the development of the market appears to be the pricing of mortality risk. A model derived[30] by DenisToplek of University of St. Gallen in Switzerland concludes that: *The results of the conducted sensitivity analysis show the substantial risk potential arising from long term projections. Actuaries need to be aware of those risks when pricing annuity contracts that cover a long time period. Moreover, these risks need to be actively managed constantly by monitoring mortality data and interest rate data. Possible tools that can help to manage this risk may be reserves that are set up using adverse scenarios or simulation techniques, for example. Other possible risk management measures may be the use of reinsurance or the transfer of longevity risk to capital markets. Currently, research as well as practitioners are strongly exploring the transfer of longevity risk to capital markets in order to create a hedging instrument for longevity risk. This paper underlines that longevity risk may be substantial and that the efforts to assess and manage this risk are necessary.*

As mentioned in the April, 2010, edition of this newsletter, JPMorgan has a division that seeks to provide a marketplace for the transfer of longevity risk, *LifeMetrics*,[31] but I regret that I cannot report on the success of this endeavour. However, there appears to be some success: it has been noted[32] *A number of recent articles have explored the important role of pure longevity insurance, i.e. a deferred life annuity that initiates payments well into the retirement years, e.g. age 80 or 85, as opposed to at the start of retirement. A collection of related products now fall under the umbrella of advanced life delayed annuities (ALDA), a.k.a. longevity insurance with a deductible. Many U.S. companies such as New York Life, Hartford Life, MetLife and Prudential Financial are now offering variants on this concept.*

Moshe Milevsky has attempted to have such a product introduced to Canada,[33] and was even able to obtain indicative quotations for such a product, but the effort failed due to institutional and regulatory barriers, of which the most laughable is the lack of a death benefit. One can well imagine a regulator or self-appointed investor advocate stridently denouncing such a useful product on the grounds that most investors would suffer a total loss of investment!

IFRS "Insurance Contracts"

The IFRS exposure draft titled "Insurance Contracts"[34] will require explicit accounting from the insurers of their hedging of long term contracts, such as annuities. Julie Dickson, head of OSFI, has decried the IFRS exposure draft in a speech[35] but minimized the potential for regulatory forbearance should the draft be approved in its current form.

The issue[36] is that when insurers purchase assets to cover their liabilities on annuities, the term of the liabilities exceeds the term of the assets, giving rise to a duration mis-match: this causes some volatility under Canadian GAAP, but the exposure draft will discount the liabilities at the risk-free government rate adjusted for a liquidity spread, rather than at the yield of the assets. This de-linking will considerably increase the volatility of earnings, which is particularly important in Canada, which uses the same set of books for earnings and capital purposes; US and UK insurers maintain separate sets of books for these purposes.

Sun Life Financial describes[37] this de-linking as its "most significant concern with the proposals". Manulife states[38] that *the proposal to use a single vector of discount rates, whether it be risk free rate plus a liquidity premium, or a high quality corporate bond rate such as a double A rate, does not help the reader of financial statements for the simple reason that spread changes or "basis risk" between the reference discount rate and the investable universe is too great*. They also point out that using any single determinant of a discounting rate (such as corporate AA bonds) will cause increased demand for that determinant, distorting the market and potentially causing increased concentration risk within the industry.

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[30] Denis Toplek, *Deferred Annuity Contracts under Stochastic Mortality and Interest Rates: Pricing and Model Risk Assessment*, University of St. Gallen, Working Paper Series in Finance Paper No. 49, May 2007, available on-line at http://www.finance.unisg.ch/org/finance/web.nsf/SysWebRessources/WP49/$FILE/WP49.pdf (accessed 2011-4-8)

[31] J.P.Morgan, *LifeMetrics*, available on-line at http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics (accessed 2011-4-8)

[32] AH. Huang, M. A. Milevsky and T.S. Salisbury, *A Different Perspective on Retirement Income Sustainability: Introducing the Ruin Contingent Life Annuity (RCLA)*, September 2007, available on-line at http://www.ifid.ca/pdf_workingpapers/WP2007SEPT15_RCLA.pdf (accessed 2011-4-8)

[33] Moshe A. Milevsky, *Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities*, Managing Retirement Assets Symposium (SoA), 2004, available on-line at http://www.ifid.ca/pdf_workingpapers/WP2004FEB_.pdf (accessed 2011-4-8)

[34] International Accounting Standards Board, *Insurance Contracts*, Exposure Draft ED/2010/8, available on-line at http://www.ifrs.org/NR/rdonlyres/508B3E26-4355-46E6-ACCF-248E76AD3457/0/ED_Insurance_Contracts_Standard_WEB.pdf (accessed 2011-4-8)

[35] Julie Dickson, *Challenges in the Life Insurance Industry*, Speech to the 2010 Life Insurance Invitational Forum, November 8, 2010, available on-line at http://www.osfi-bsif.gc.ca/app/DocRepository/1/eng/speeches/jd20101108_e.pdf (accessed 2011-4-8)

[36] BMO Capital Markes Equity Research Financial Services, *Weekly Update*, August 2, 2010, available on-line at http://www.ifrs.org/NR/rdonlyres/7CC66D61-28F8-47C7-A33C-002BA1AB0756/0/CL133CanAnalysts.pdf (accessed 2011-4-9)

[37] Colm Freyne, CFO Sun Life Financial, *Comment Letter RE: Exposure Draft – Insurance Contracts*, 2010-11-30, available on-line at http://www.ifrs.org/NR/rdonlyres/3B0B20BC-A0DD-4B74-B088-DB60695AC1C7/17895/20101130211106_sunlifefinancialincresponseletteroninsurancecontracts.pdf (accessed 2011-4-9)

[38] Lynda Sullivan, Executive Vice President and Controller, Manulife Financial Corporation, *Comment Letter Re: Insurance Contracts*, 2010-11-30, available on-line at http://www.ifrs.org/NR/rdonlyres/52E1E098-5BB6-4A48-8753-D7B8FFACFAF7/17907/20101130231112_MFCResponsetoIFRS4ED_30Nov2010_.pdf (accessed 2011-4-9)

However, I am yet to be convinced of the adequacy of these arguments against the plan. The insurers claim that changes in credit risk are properly accounted for; but if this is the case, then the remainder of the change is changes in the liquidity premium[39] I confess I don't understand why this liquidity premium cannot be made explicit, to be disclosed and used in the financial statements.

It remains too early to tell what the outcome of this dispute may be, but there is the potential for annuity prices to increase if the proposal is adopted as Canadian insurers seek to increase the profitability of annuities to offset the greater volatility in their capital requirements that will result.

Inflation and Price Level Targetting

One of the great uncertainties in investing in an annuity – or in fixed income of any kind – is the future course of inflation. Many of those in or approaching retirement will have memories of the 1970's and early eighties, when inflation ravaged the land and newspapers were filled with stories of seniors on fixed incomes finding their standard of living drastically reduced.

When a government wishes to spend money, it has three – and only three – options:

- Tax it
- Borrow it
- Print it

It should be clear that only the first option is sustainable in the long run, although I have no problems with the Keynesian idea that deficits are perfectly acceptable during recessions – provided they are paid back with surpluses in good times, which is the hard part. The most destructive option, by far, is to print it (nowadays done by book entry, with the central bank simply declaring that the government has a balance of X), although this too can be acceptable in the short term.[40]

The question is, how should the Central Bank seek to control inflation?

We have become familiar with Inflation Targetting, whereby the *Bank of Canada aims to keep inflation at the 2 per cent target, the midpoint of the 1 to 3 per cent inflation-control target range*.[41] This has been *the cornerstone of monetary policy in Canada since 1991*, when the Canadian inflation rate was about 5%.[42]

However, the bank has been researching the potential for moving to a regime of Price Level Targetting since 2006.[43] The difference can be explained succinctly:[44] *Despite its recent successes in terms of macrostabilization, several authors have highlighted some shortcomings in the inflation-targeting (IT) framework. Most notably, uncertainty on the price level grows with the planning horizon, since central banks with inflation targets accommodate shocks to the price level, taking the post-shock level as given and aiming to stabilize inflation from this level. In fact, the price level is unbounded at very distant horizons. Price-level targeting (PT) mitigates this uncertainty by committing central banks to restore the price level to a preannounced target following shocks. PT is frequently described as a departure from IT's prescription for letting "bygones be bygones."*

It will be noted that the HIMI implementation of the retirement calculator (and all others, as far as I know) implicitly assume Price Level Targetting as the basis for their treatment of inflation: the long-term rate is always equal to the short-term rate. A more realistic model of inflation in the calculator under an Inflation Targetting regime would allow inflation to vary within the defined bounds (and sometimes outside of them!) without being influenced by prior experience – so the potential adoption of Price Level Targetting will, beyond its broader policy implications, make life easier for manufacturers of financial calculators!

Annuities and Fixed-Income in Retirement Planning

It has been pointed out[45] that *The main reason retirement portfolios don't survive is that they are endangered when payouts exceed dividend and interest income. When that happens, shares or bonds must be sold. Do that in a down market, and the portfolio is damaged. That's why Peter Lynch's 1995 advice – that investors could safely withdraw 7 percent a year from a 100 percent stock portfolio – was dead wrong.*

So how do we increase portfolio survival?

Other research has shown two options. Ibbotson Associates has put its seal of approval on REITs as a portfolio diversification tool. (Think about it: REITs generally have more current income than stocks.) Other research has shown that adding a life annuity to your retirement portfolio can enhance the odds that your overall portfolio will survive. (Again, life annuities provide current cash income and reduce the need to make withdrawals from the remainder of your portfolio.

This is particularly interesting in light of the calculations discussed in the December, 2010, edition of this newsletter which examined the credit quality of split-share corporations. In that article, one of the issues examined was the importance of the dividend yield of the underlying portfolio to the credit quality of the corporation's preferred shares: it was found *that the effect of keeping the dividend constant while increasing return is about the same as keeping return constant, but increasing the proportion paid as dividend!*

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39 See http://www.himivest.com/media/moneysaver_0907.pdf

40 This is the idea behind Quantitative Easing, which has been deemed to have achieved its objectives so far by Joseph Gagnon, Matthew Rasking, Julie Remache and Brian Sack, *Large-Scale Asset Purchases by the Federal Reserve: Did They Work?*, Federal Reserve Bank of New York Economic Policy Review (forthcoming), available on-line at http://www.newyorkfed.org/research/epr/forthcoming/1104gagn.pdf (accessed 2011-4-9). For a variety of perspectives, see the links at *QE2 and Inflation*, PrefBlog, available on-line at http://www.prefblog.com/?p=13175

41 Bank of Canada, *Monetary Policy: Inflation*, Bank of Canada, available on-line at http://www.bankofcanada.ca/en/inflation/index.html (accessed 2011-4-9)

42 Bank of Canada, *Inflation Control Target*, Backgrounder, September 2009, available on-line at http://www.bankofcanada.ca/en/backgrounders/bg-i3.html (accessed 2011-4-9)

43 John Murray, *Research on Inflation Targeting*, Bank of Canada, Review, Spring 2009, available on-line at http://www.bankofcanada.ca/en/review/spring09/review_spring09.pdf (accessed 2011-4-9)

44 Robert Amano, Tom Carter and Don Coletti, *Next Steps for Canadian Monetary Policy*, Bank of Canada, Review, Spring 2009, available on-line at http://www.bankofcanada.ca/en/review/spring09/review_spring09.pdf (accessed 2011-4-9)

45 Scott Burns, *Portfolio Survival: Income Trumps Diversification*, Asset Builder, June 1 2004, available on-line at http://assetbuilder.com/blogs/scott_burns/archive/2004/06/01/Portfolio-Survival_3A00_--Income-Trumps-Diversification.aspx (accessed 2011-4-5)

Retirement Withdrawal Planning

The problem of cash withdrawals requiring the sale of investments discussed above is the basis of the 'Sequence of Returns Risk' often discussed by Dr. Milevsky[46] but has been highlighted before by even more highly esteemed pundits.

William J. Bernstein pointed out[47] *Most of you have seen the nifty retirement software available from the likes of Vanguard and T. Rowe Price which provides the mathematical muscle to help you plan your retirement. Input your retirement age, expected lifespan, required annual income, rate of inflation and investment return, and hey presto, you find out that to avoid a golden years diet of Alpo you need the GDP of the average Central American republic.*

Problem is, it may quite possibly be worse than that. These calculators all make the same erroneous assumption – that your expected rate of return is the same each and every year. In other words, let's assume that the real (inflation adjusted) rate of return of the S&P 500 will be 7% in the future. You might conclude that you can withdraw an inflation adjusted $70,000 of your $1,000,000 Vanguard Index Trust 500 IRA each and every year indefinitely, and maintain yourself with the same real income in the long run. And you'd be wrong.

It turns out that if you have rotten returns in the first decade you will run out of money long before reversion to the mean saves your bacon in later years.

In a follow-up article[48] he introduced Monte Carlo analysis and derived "success rates" shown in Tables A2 and A8:

Table A2: 30-Year Success Rates (Monthly Withdrawals) Real Returns: Stocks 7%, Bonds 2.5%, correlation 0% [from Bernstein]

	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	7.0%	12.0%	98.7%	93.4%	81.0%	63.3%
75/25	6.0%	9.09%	99.3%	93.4%	76.0%	50.3%
50/50	4.9%	6.5%	99.6%	91.4%	61.2%	24.9%
25/75	3.8%	4.8%	99.3%	77.4%	24.8%	2.6%
Bonds	2.5%	5.0%	87.2%	33.4%	3.7%	1.3%

Table A8 : 30-Year Success Rates (Monthly Withdrawals) Returns: Stocks 4.5%, Bonds 3.5%, correlation 0% [from Bernstein]

	Return	SD	4.0%	5.0%	6.0%	7.0%
Stocks	4.5%	12.0%	88.8%	70.4%	48.0%	28.3%
75/25	4.4%	9.09%	94.3%	75.2%	46.6%	22.2%
50/50	4.2%	6.5%	98.2%	80.1%	41.3%	12.0%
25/75	3.9%	4.8%	99.5%	81.1%	28.9%	3.4%
Bonds	3.5%	5.0%	98.2%	68.3%	18.5%	0.2%

One of the problems with this type of calculation is the binary nature of the objective function: the simulation can only succeed or fail, with success being defined as an account balance of greater than zero at the end of the simulation.

While this may be appropriate for some, there are many who also aim to leave legacies after their demise; but assigning appropriate weights to the end-value relative to the chance of shortfall is a very personal question usually not discussed.

However, as pointed out in my previous discussion of annuities,[49] a binary objective function simply fails to take into consideration the uncertainties inherent in long-term retirement planning: *Dead and alive are not the only states and retirees do not, as a rule, drop dead immediately after drawing the losing ticket in the mortality pool. It is not unusual for a senior to be told of a degenerative disease; of impending blindness; or for other medical problems to be diagnosed that drastically alter his planning horizon. In these circumstances, the ability to shift consumption into the period in which it can be best enjoyed has immense value, and this change in consumption timing is not possible with an annuity.*

[46] Moshe A. Milevksy, *Feast or Famine First*, www.researchmag.com, available on-line at http://www.ifid.ca/pdf_newsletters/PFA_2007DEC_FEAST.pdf (accessed 2011-4-10)

[47] William J. Bernstein, *The Retirement Calculator from Hell*, 1998, available on-line at http://www.efficientfrontier.com/ef/998/hell.htm (accessed 2011-4-5)

[48] William J. Bernstein, *The Retirement Calculator From Hell – Part II*, 2001, available on-line at http://www.efficientfrontier.com/ef/101/hell101.htm (accessed 2011-4-5)

[49] This newsletter, April 2010.

Retirement Calculator: Features of Pritz, Van and Ponzo (PVP) Implementation[50]

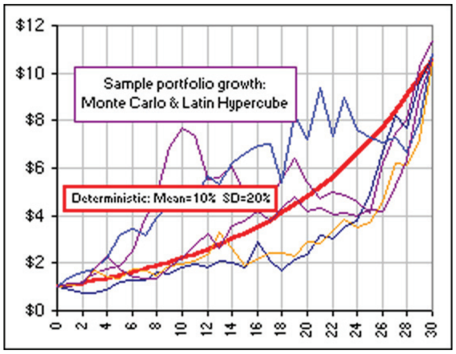
Retirement calculators have been around for a long time – and with the introduction of personal computers and the embedding of simple programming capabilities in spreadsheets, their use has exploded. In specifying the HIMI implementation of a calculator, I examined other approaches; I will look at the Pritz, Van and Ponzo implementation and its features in this section.

Latin Hypercube Sampling (LHS): This sampling technique orders the randomized returns and then subdivides them into a series of intervals, each containing a set number of points. Subsequent sampling in the Monte-Carlo simulations then ensure that one point is selected from each interval.

The pros and cons of this sampling technique have been discussed by Peter Ponzo[51] who points out that *With Latin Hypercube sampling, our 30 annual returns hardly change from one possible 30-year Monte Carlo future to the next. It's just the ordering of these returns which changes … and that would certainly affect the result if there were annual deposits or withdrawals. However, with just our $1 investment, the result of applying 30 annual returns, in random order, yields the same final portfolio … we'd expect to get wildly different portfolios during the 30 years, but only a slight variation in the final portfolio.*

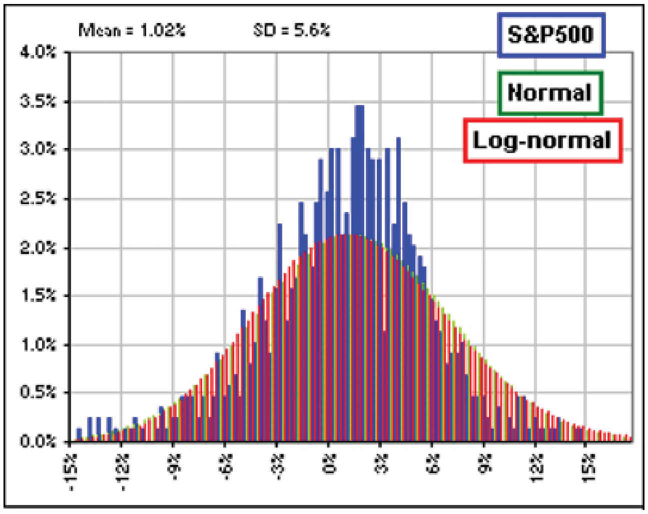
Chart A-1 shows the convergence of portfolios to a similar final value when LHS is used in the Monte Carlo simulation. There will doubtless be many learned arguments possible regarding the strengths and weaknesses of LHS, but I am leaving this option out of my simulator on the basis that it is the outliers which are of fundamental interest to us – in other words, that percentage of completely randomized samples which result in portfolio ruin.

A-1



Log-Normal or Normal Returns: Normal and log-normal distributions have also been discussed extensively by Dr. Ponzo.[52] With respect to the results of this particular spreadsheet, he remarks[53] with respect: *the Normal and Log-normal distributions are almost identical which makes one wonder why the big fuss over whether to choose one over t'other.*

A-3



50 Described and published by Peter Ponzo at http://www.financialwebring.org/gummy-stuff/Monte-Carlo.htm (accessed 2011-3-26)

51 Peter Ponzo, *Latin Hypercube and sampling stuff*, available on-line at http://www.financialwebring.org/gummy-stuff/latin-hypercube.htm (accessed 2011-3-26)

52 Peter Ponzo, *Normal vs. Log-normal*, available on-line at http://www.financialwebring.org/gummy-stuff/normal_log-normal.htm (accessed 2011-3-26)

53 Peter Ponzo, *Monte Carlo Simulation: a continuation of part I*, available on-line at http://www.financialwebring.org/gummy-stuff/Monte-Carlo-2.htm (accessed 2011-3-26)

I will agree that log-normality has the satisfying property that investment returns cannot be worse than -100%, whereas this is possible with normally distributed returns. However, given the relatively small range of monthly returns, the chance of such a return is minimal and I contend that the ease of understanding and simplicity of programming outweighs any such difference.

I will also assert that the importance of skew and kurtosis in 'real world' returns outweighs any difference between the two distribution methodologies and that the effect of our assumptions regarding the future mean and standard deviation of the returns completely swamps them.

Thus, returns distributions in my implementation of the spreadsheet will be Normal, not log-Normal – but I feel that the best solution is to use historical results of a chosen security as the controller of the distribution, rather than a mathematical formula.

Equity Returns (From Historical): The PVP implementation calculates portfolio values by year. There are 642 months of TSE data available, and 905 months of S&P 500 data. A random month is chosen, twelve or more months prior to the end of the data, and a total return for the next twelve months calculated and used in the spreadsheet.

Dr. Ponzo does not explain the rationale behind this implementation. It is possible that it is done to account, at least partially, for sequential correlation of monthly returns (e.g., up months followed by down months); it is also possible that it was chosen for computational speed and convenience. My implementation will select returns for each month randomly, without regard for the other selections.

Equity Returns (From Formula): The PVP implementation also allows selection of a normal distribution. This option will also be available in my implementation.

Bond Returns: If the TSE data is used as the source of equity returns, then the bond return is defined as the return for Canadian long bonds for the corresponding months. However, it is not clear whether these data are for Government bonds, corporates, or the overall long-term index.

I do not like this selection methodology on several grounds:

- The wildly varying inflation rates over the past 642 months (over fifty years!) means that the future relationship between stock and bond returns may no longer reflect the past. For example, the spreadsheet data associates a monthly return of 7.3% for stocks with a return of -4.8% for bonds.
- The correlation of the monthly historical data is -0.16.
- If we buy a 30-year bond then we know – barring default – what the thirty year return is going to be, regardless of month-to-month price fluctuations.

If S&P data is used for equity returns in the PVP implementation, then the bond return is uncorrelated with the stock return.

If equity returns are derived formulaically, the bond return is calculated with the desired correlation in accordance with the formulae:

G1 = SR + (e1 * Sqr(1 – cor * cor) + e2 * cor) * SV

G2 = BR + e2 * BV

Where:

G1 is the return on equities

SR is the expected annual average return on stocks

SV is the expected annual average SD on stocks

BR is the expected annual average return on bonds

BV is the expected annual average SD on bonds

e1 and e2 are that year's variance from the mean, expressed in terms of Standard Deviation. These values are randomly chosen from 400 possibilities, evenly distributed in probability space.

cor is the correlation between the returns on the two asset classes

Sqr(x) = x^2

This rather elegant method of constructing correlated sets of data is specified in many sources;[54] I'm sure it has a name, but don't know what it is.

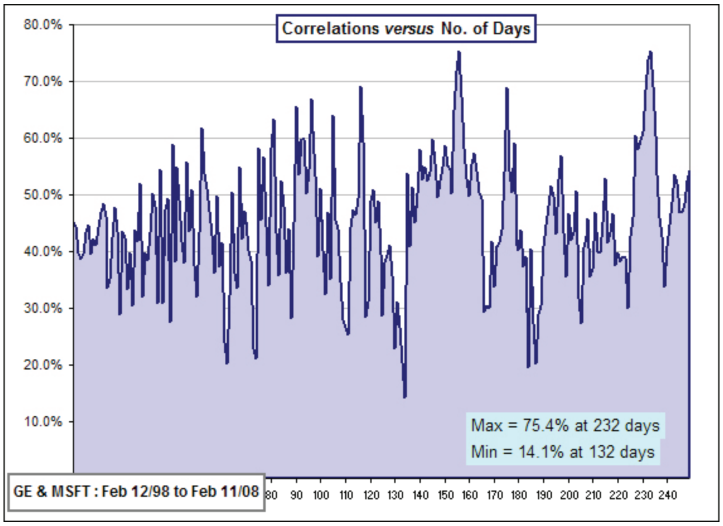
One implication of the above is that the correlation is being applied to annual returns, whereas most correlations available are calculated using monthly data. As always, it must be remembered that correlations in capital markets are not very robust; there can be significant differences in correlation depending upon the sampling frequency of the data[55] and on the economic conditions at the time of data collection.[56] As is always the case with long-term macro forecasting, assumptions – including future levels of correlation – should be very general and results taken with a grain of salt, as discussed in the October, 2009, edition of this newsletter.

[54] Peter Ponzo, *Correlations and Pearson Coefficient*, available on-line at http://www.financialwebring.org/gummy-stuff/correlation_coefficients.htm (accessed 2011-3-28). I have also seen the formula on the MATLAB newsgroup, *Thread Subject: Generation of correlated random numbers*, available on-line at http://www.mathworks.com/matlabcentral/newsreader/view_thread/243959 (accessed 2011-3-28).

[55] Peter Ponzo, *Correlation and Time Periods*, available on-line at http://www.financialwebring.org/gummy-stuff/correlation-vs-days.htm (accessed 2011-3-28)

[56] See the September 2009 edition of this newsletter; also, note that the Kansas City Financial Stress Index uses a change in the sign of correlation between Treasuries and equities as an indicator of financial stress; Craig S. Hakkio and William R. Keeton, *Financial Stress: What Is It, How Can It Be Measured, and Why Does It Matter?*, Federal Reserve Bank of Kansas City, Economic Review: Second Quarter 2009, available on-line at http://www.kansascityfed.org/PUBLICAT/ECONREV/pdf/09q2hakkio_keeton.pdf (accessed 2011-3-28)

A-2



Retirement Calculator: HIMI Implementation (Static, 2011-4-8)

This calculator is available on-line at http://www.prefblog.com/xls/retirementWithdrawals.xls. It must be downloaded in order to run, as it is an Excel spreadsheet with attached code running under Microsoft Visual Basic for Applications (which is supplied with Excel).

Annualization in the Presence of Volatility

In accordance with the formula[57]:

Annualized Return = Average Return – (1/2) Volatility^2

The "Characterize Data" functionality has been improved; the prior implementation in http://www.prefblog.com/xls/splitShareCreditQuality.xls discussed in the December, 2010, edition of this newsletter, did not account for the effect of the "variance drag".

Thus, when Characterizing Data using the template XFN.TO for the period 2002-12-8 to 2010-12-8, the effect of the adjustment is shown in Table A1:

Table A1: Effect of Variance Drag Adjustment

Parameter	Without Correction	With Correction
Expected Return (input)	7.00%	7.00%
Mean Return (one year)	8.46%	8.06%
Median Return (one year)	7.40%	7.32%

Adjustment of Dividend and Coupon Rates

In the similar spreadsheet which examined the credit quality of SplitShare preferreds,[58] I noted that *it will be required that the model used by the spreadsheet holds dividends constant; it is only the price component of total return that compounds over time. The lack of sophistication in the spreadsheet is excused by its intended purpose of investigating the credit quality of SplitShare preferreds, most of which will have a term to maturity of five years or less.*, which is no longer a reasonable assumption for the 30-year retirement withdrawal calculator.

However, I do not wish to make dividends increase in lockstep with total return; although it doesn't make any difference to results in the current implementation, it could do so in the future. Accordingly, the dividend yield is checked every year: if the yield is more than 1% different from the initial yield (in absolute terms; e.g., if the initial yield is 3.5% then the trigger yields are 2.5% and 4.5%), then the dividend adjusts by 0.5% towards the target.

57 Peter Ponzo, *Returns and Volatility*, available on-line at http://www.financialwebring.org/gummy-stuff/returns-volatility.htm (accessed 2011-3-31)

58 See the December, 2010, edition of this newsletter. The spreadsheet is available at http://www.prefblog.com/xls/splitShareCreditQuality.xls

Price Approaches Zero

A feature (not a bug!) of the Hymas Implementation is that the prices of either bonds or stocks can approach zero, due to the cumulative effects of poor returns combined with the drag involved in paying dividends or coupons.

When the price of either asset declines below $0.10 (from its initial value of $100) the number of units held is reset to zero (you now own worthless paper!) and the price is reset to $100.00 and recommences varying as the simulation run continues.

This feature is dictated by the internal logic of the spreadsheet; early versions which did not have this check-and-reset algorithm gave very strange results due to stocks sometimes having a negative value.

Other implementations do not require this adjustment, as they focus on total return of the portfolio and do not explicitly consider dividends.

Rebalancing

William Bernstein discusses the Rebalancing Bonus in an excellent article,[59] concluding that *the intrinsic rebalancing potential of any asset pair is the difference between its mean variance and covariance*, which can be expressed as a formula

$$RB_{1,2} = X_1X_2(Var_1/2 + Var_2/2 – Covar_{1,2})$$

Where: $RB_{1,2}$ is the Rebalancing Bonus for a portfolio comprised of assets 1 and 2, defined as the excess return of a rebalanced portfolio over a non-rebalanced portfolio

X_1 is the amount of X_1 held

X_2 is the amount of X_2 held

Var_1 is the variance of returns of asset 1, with returns measured over a rebalancing period

Var_2 is the variance of returns of asset 2, similarly to above

$Covar_{1,2}$ is the covariance between assets 1 and 2

Most implementations of Retirement Withdrawal Calculators implicitly assume annual rebalancing; after some experimentation with a different scheme which attempted to minimize notional transaction costs, the HIMI implementation follows them.

Treatment of Bonds

Bonds are a problem and I have not yet seen a retirement calculator that deals with them adequately; however, I have sketched out a potential solution in the section on future enhancements, below.

The great advantage of fixed income is quite simple: the income is fixed. Barring default, an investor will receive exactly the same amount of cash each and every year until maturity, at which point his capital will be returned.

The ideal fixed income investment is therefore a perpetual – assuming, of course, that in addition to barring default we also bar inflation. If an investor can finance his cash needs through the coupons on a perpetual bond, then he doesn't care that the price of this instrument is extremely volatile.

It is for this reason, as well as the ability to hedge annuities (as described above) that I favour longer-term fixed income investments for retirees, to the horror of most financial planners who have a predilection for shorter-term investments. Inflation is always a worry, of course, but one should not be looking for inflation protection in a fixed income portfolio – that's a contradiction in terms! Inflation protection is the responsibility of the equities portion.

However, this constancy of return is difficult to work into standard models, including the first iteration of my own implementation. It is with a heavy heart that I follow the crowd and assume that bond returns vary with a normal distribution, with each year's return uncorrelated with the previous years. This leads to results that are not internally consistent, as described below in the section Effect of Increasing Bond Variance.

However, I will recommend that a relatively low variance be used with the model. While this does not capture the year-over-year variation in bond values (particularly in the first years of a simulation of the purchase of a thirty year bond), it does have the desirable property of keeping the long term returns within reasonable bounds.

59 William J. Bernstein, *The Rebalancing Bonus: Theory and Practice*, 1996, available on-line at http://www.efficientfrontier.com/ef/996/rebal.htm (accessed 2011-4-6)

Characteristics of the HIMI Static Draft Retirement Withdrawal Model

How Many Runs Are Necessary?

It will not take long for users to discover that a full run of 8,191 simulations takes a considerable amount of time. Why so long? My first instinct is to growl that it's because it's not a toy. More diplomatically expressed reasons are that the spreadsheet uses Microsoft's Visual Basic for Applications, which is not only a rather slow programming language, but one that I rarely use – so there may be parts of the code that are badly written from a programmer's perspective. Additionally, I have not spent a lot of programming time optimizing for speed; one way or another, there are probably many ways to speed execution of the programme while maintaining functionality.

That being said, there's no point in doing too much work. Given the fact that our assumptions are imprecise, there is very little to be gained by performing simulations until the 6th decimal place of our final values is determined precisely. Accordingly, a full simulation of 8,191 runs was performed according to the specifications shown in Table A4, which will henceforth be referred to as the "Initial Case". Note that the annuity rate of $630 monthly per $100,000 invested has been estimated for a 65 year old male, with no guaranteed pay-out period.[60]

Table A4: Initial Case Simulation Settings

Setting	Value
Variance Template	XFN.TO from 2002-12-8 to 2010-12-8
Expected Annualized Return	7.00%
Dividend Yield	3.50%
Bonds Expected Return	5.50%
Bond Coupon	5.50%
Bond Standard Deviation	0.5%
Annuity Rate	630.00
Correlation Stocks-Bonds	0.50
Inflation	2%
Stocks Allocation	50%
Bond Allocation	50%
Annuity Allocation	0%
Initial Withdrawal Rate	4.00%

After 8,191 simulations, results are reported as shown in Table A5. The probability of ruin is 3.7%.

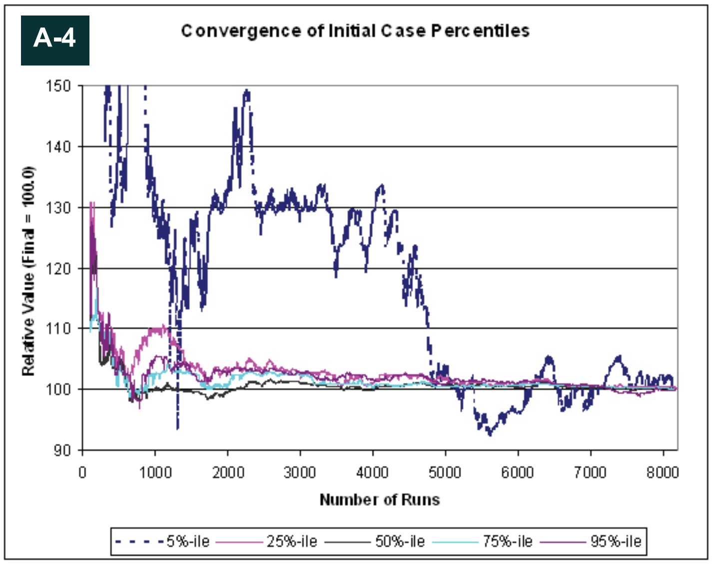
Table A5: Results of Initial Case Simulation

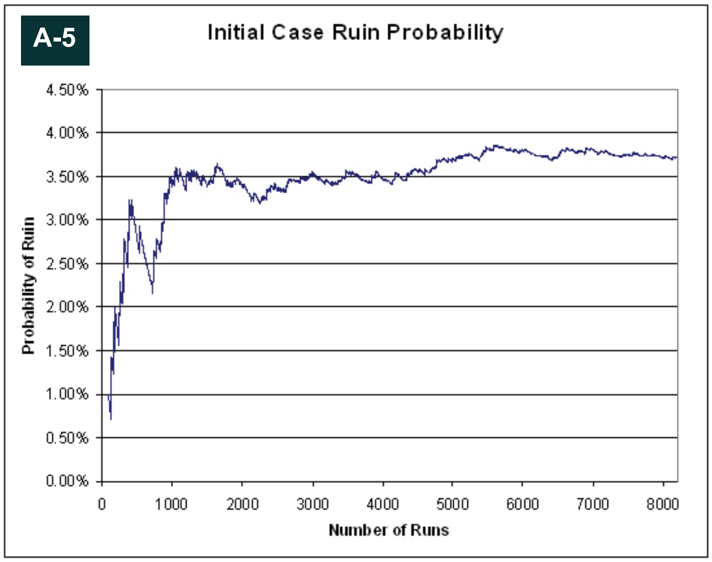
Year	5%-ile	25%-ile	50%-ile	75%-ile	95%-ile
5	812,000	978,000	1,124,000	1,283,000	1,552,000
10	735,000	1,023,000	1.267,000	1,556,000	2,098,000
15	656,000	1,067,000	1,431,000	1,886,000	2,779,000
20	535,000	1,104,000	1.623,000	2,299,000	3,702,000
25	359,000	1,123,000	1,856,000	2,827,000	4,934,000
30	97,000	1,103,000	2,142,000	3,511,000	6,618,000

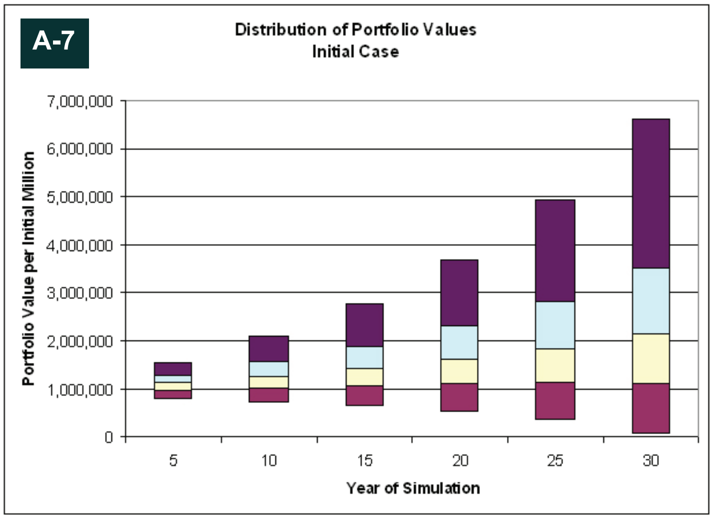
[60] Determined 2011-4-6 from Canadian Business, *Annuity Rates – Single Life Male No Guarantee*, data provided by Cannex, available on-line at http://www.canadianbusiness.com/my_money/rates/annuity_single_life_male_no_guarantee/index.jsp (accessed 2011-4-6)

Charts A-4 and A-5 show the convergence of values to the final result. There does not appear to be much value in running more than 2,000 simulations, so that will be the number used in all other simulations discussed in this essay.

Chart A-7 shows the distribution of values at various terms during the simulation, with the data taken from Table A-5.







Improving the Initial Case

Table A7 shows my successive steps to an optimal portfolio, as I try to maximize the withdrawal rate while keeping the portfolio reasonably constant in terms of expected terminal value and attempting to minimize the ruin rate.

Table A7: Successive Steps to an Optimal Portfolio [Settings not specified are as for the Initial Case, Table A4]

Stocks	Bonds	Annuities	Initial Withdrawal Rate	Ruin Rate	Median 30-Year value
50%	50%	0%	4.0%	3.7%	2,142,000
50%	50%	0%	4.5%	11.0%	1,614,000
50%	50%	0%	5.0%	20.1%	1,080,000
45%	55%	0%	5.0%	18.2%	1,056,000
45%	50%	5%	5.0%	17.6%	997,000
40%	50%	10%	5.0%	15.5%	1067,000
35%	50%	15%	5.0%	12.4%	1,032,000
30%	50%	20%	5.0%	10.6%	974,000
25%	50%	25%	5.0%	6.1%	1,000,000
20%	50%	30%	5.0%	3.0%	944,000
25%	45%	30%	5.0%	5.5%	1,024,000

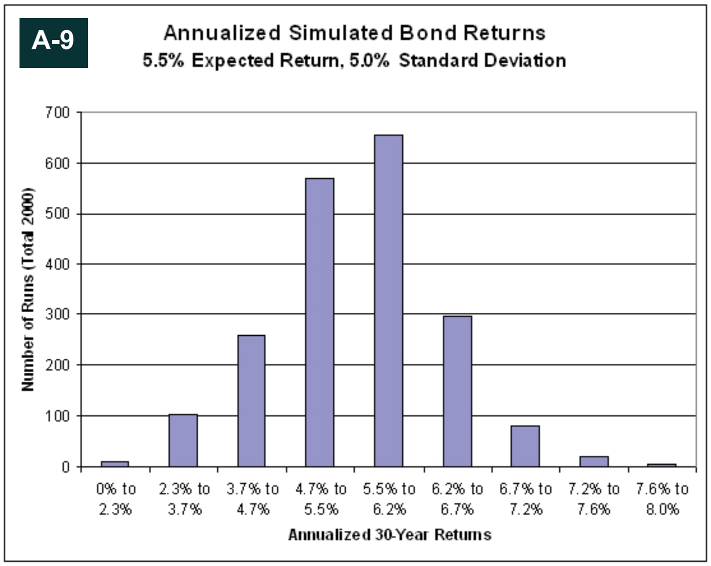
Effect of Increasing Bond Variance

Table A9: Effect of Increasing Bond Variance [Settings not specified are as for the final case of Table A4]

Stocks	Bonds	Annuities	Initial Withdrawal Rate	Bond Standard Deviation	Ruin Rate	Median 30-Year value
25%	45%	30%	5.0%	0.5%	5.5%	1,024,000
25%	45%	30%	5.0%	1.0%	5.4%	975,000
25%	45%	30%	5.0%	1.5%	7.2%	1,022,000
25%	45%	30%	5.0%	2.0%	8.9%	961,000
25%	45%	30%	5.0%	2.5%	8.0%	983,000
25%	45%	30%	5.0%	3.0%	9.0%	1,000,000
25%	45%	30%	5.0%	3.5%	9.4%	938,000
25%	45%	30%	5.0%	4.0%	9.1%	1,037,000
25%	45%	30%	5.0%	4.5%	11.2%	1,011,000
25%	45%	30%	5.0%	5.0%	16.0%	955,000
30%	40%	30%	5.0%	5.0%	14.0%	997,000
35%	35%	30%	5.0%	5.0%	14.8%	1,044,000
35%	40%	25%	5.0%	5.0%	16.5%	1,081,000

There's only one problem with the figures I have so laboriously calculated and entered into Table A9: they're nonsensical. We can recover the bond total return factors for each run of the simulation from the spreadsheet's "Statistics" tab, column "R", and convert these returns into annualized percentage yields by taking the 30th root of these numbers. Results are shown in Chart A9.

The longer the term of the bond, the longer its coupon will be "locked in" – if a thirty-year bond is selected at the commencement of the exercise, as I assume when setting my Initial Case variance at 0.5%,the a return in excess of 5.5% is impossible and a return of less than 5.5% requires the assumption of default.



It might be argued that shorter-term bonds could be chosen, with less violent changes in market interest rates being required to explain the data as the term shortens – but then the initial assumption of 5.5% coupon rate becomes very difficult to justify, given the assumption of a constant 2.0% inflation rate.

Clearly, if one wants to incorporate bonds into a retirement withdrawals calculator in a manner that does not give offense to logic, then a more sophisticated approach is required; a potential enhancement is discussed below. In the meantime, it is interesting to note that as the variance increases, the probability of ruin also increases without significantly affecting the median value at the end of a 30-year run. Additionally, results are slightly improved by reallocating assets from bonds to stocks. This suggests that most retirement calculator methodologies - which in all implementations I have seen treat bonds in this manner with variance on or beyond the higher end of the scale used in Table A9 - recommend an unjustifiably low allocation to bonds.

Jack & Jill: A Case Study

The case study of Jack & Jill at the closing of the Milevsky & Macqueen book referenced earlier is very difficult to follow in specific terms, but there are some nuggets of information.

The new retirees were able to purchase an inflation-adjusted annuity with an initial annual income $12,800 for $200,000. Additionally, their portfolio of 60% stocks/40% bonds is expected to produce real returns of 3.68%.

To be perfectly frank, these numbers look as if they have been selected to enhance the attractiveness of annuitization. According to current rates[61], a Joint Life No Guarantee Registered Annuity, with 65-year-old annuitants, will pay about $500 monthly per $100,000 invested (non-registered will pay fractionally less) – and this is a nominal annuity, not a gold-plated inflation adjusted one!

It is more difficult to assess the 3.68% portfolio real-return, but if we assume a 5% real return on equities, a real return of 1.7% on bonds is implied and, what's more, it is so exactly implied that I suspect that these are in fact the numbers used. At current rates, this will imply that the bond return used as the model is 3.7%, which is the yield on long-term nominal Canadas.[62] Not only is an investment in long-term Canadas a sub-optimal strategy for a buy-and-hold investor (since one does not capture any liquidity premium), it is generally agreed that government bond yields are still at recessionary levels due to lingering effects of the Credit Crunch and spill-over from Quantitative Easing.

However, the volatility of the asset classes is not specified, nor is the planning horizon of the calculation. A further complaint about the level of detail in the book is that a terminal value of $107,000 is claimed from the initial portfolio value of $850,000 (of which $650,000 is invested in the capital markets), but this drastic decline in portfolio value seems to be accomplished by maintaining the probability of ruin at less than 25%.

All this sounds very odd. It is quite certain, of course, that there is a set of assumptions that will produce these results – but I suspect that these assumptions are very far removed from what I consider to be the reasonable assumptions made explicit in this essay.

Be that as it may, I thought it would be fun to attempt an optimization of the situation using the methodology and assumptions used elsewhere in this essay; results are shown in Table A10.

61 See http://www.canadianbusiness.com/my_money/rates/annuity_joint_life_no_guarantee_r/index.jsp

62 Bank of Canada, *Interest Rates*, available on-line at http://www.bankofcanada.ca/en/rates/bonds.html (accessed 2011-4-9)

Table A10: Successive Steps to an Optimal Portfolio [Settings not specified are as for the Initial Case, Table A4]					
Stocks	Bonds	Annuities	Initial Withdrawal Rate	Ruin Rate	Median 30-Year value
60%	40%	0%	6.1%	51%	-1,000
55%	45%	0%	6.1%	50%	0
50%	50%	0%	6.1%	51%	-2,000
50%	45%	5%	6.1%	51%	-2,000
45%	45%	10%	6.1%	54%	-7,000
40%	45%	15%	6.1%	52%	-3,000
35%	45%	20%	6.1%	51%	-1,000
30%	40%	30%	6.1%	54%	-4,000
25%	35%	40%	6.1%	51%	-1,000
5%	5%	90%	6.1%	50%	0

So it appears that using the reasonable and explicit assumptions of the calculator, we are not able to stave off ruin for Jack and Jill – although it must be remembered that even in the case of formal ruination, they will still have income from whatever annuity they purchased.

Future Enhancements to the HIMI Implementation

A quant's work is never done – and no, it's not because we're too busy playing video games instead! Any model will involve various assumptions, and any set of assumptions will have a weak point and, in most cases, it will possible to find certain situations in which the assumptions lead to a clear error. In developing and experimenting with the model, from the perspective of an investment counsellor seeking to recommend portfolios that will assist clients to achieve financial objectives, several enhancements have occurred to me; some of these are described in this section.

Future Enhancement: A Dynamic Model

The main weakness of the Static Model is simply that it is static. The proportions of stocks, bonds and annuities are fixed at the commencement of the exercise, and while they can vary due to the effect of disproportionate returns and cash-flows, the target remains the same throughout the entire thirty year period of the simulation, regardless of the actual withdrawal rate (which will change, since the withdrawn amount increases by inflation every year, whereas the portfolio value will vary considerably).

An investor who decided that a 3% withdrawal rate and 80% stock composition was prudent at the age of 65 might think differently after a decade of extremely poor stock returns leaves him with a 9% withdrawal rate at age 75!

One very clear example of this is illustrated in Chart A-7, shown above in the section "How Many Runs Are Necessary?" In this essay I have been focusing on the ability of each asset allocation to survive 30-years (if for no other reason than that it's standard), but interim values are very important.

Consider the distribution at 15 years! The median range of returns (from the 25th to the 75th percentile) spans a range of over $800,000 (as specified in Table A5). In the 15th year, the initial withdrawal rate of $40,000 p.a. has inflated to about $53,000 p.a. – so the range of withdrawal percentages corresponding to the asset values is now in a range of 2.8% to 5.0%. It seems most unreasonable to suppose that the actions of the investors are the same in either case – to continue with the plan they commenced 15 years prior. It will be noted that the 5.0% rate after 15 years has every chance of being sustainable for the next 15 years – but what if results had been a little worse and the withdrawal rate a little higher? Would that not be an indication that the investors might wish to review their plan, and perhaps take advantage of the fact that annuity payout rates have improved by the accumulation of 15 years of mortality credits?

A proper retirement withdrawal planner will take account of this and adjust portfolio composition annually, shifting the percentage allocation to each asset class to account for changing conditions. The problem is: How do we determine the shifts?

I propose that a Dynamic Model may be planned as a two-dimensional grid: each planning horizon and withdrawal rate will be assigned a particular asset allocation, as shown in its initial form in Table A3, which corresponds to the Static Model, in which a constant asset allocation is chosen regardless of remaining term of the simulation or what the withdrawal rate has become.

Table A3: Initial Asset Allocation Grid for Dynamic Model

Withdrawal Rate	Planning Horizon				
	1 Year	2 Years	……	29 Years	30 Years
1%	50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity	…	50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity
1.5%	50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity		50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity
….	…	…	…	…	…
9.5%	50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity		50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity
10.0%	50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity		50% Stocks 50% Bonds 0% Annuity	50% Stocks 50% Bonds 0% Annuity

Additionally, an objective function is needed in order to determine whether one particular setting is better than another; this should incorporate both risk and return. For instance, we might say that we'll determine "risk" as the 5th percentile of the simulation, while "reward" is the 50th percentile (which is the median expected terminal value); and we could say that the former is three times as important as the latter, so our objective function is:

O = 3*(5th Percentile) + (50th Percentile)

An example of how this can be used to differentiate between settings is shown in Table A4.

Table A4: An Example of Objective Function Scoring of Different Simulation Settings

	5th Percentile	50th Percentile	Objective Function Value
Setting A (most risk, most return)	$50,000	$1,500,000	1,650,000
Setting B (medium risk, medium return)	$100,000	$1.400,000	1,750,000
Setting C (high risk, high return)	$125,000	$1.300,000	1,675,000

According to the Objective Function and the data in Table A4, Setting "B" represents the best mix of risk and return, so in future runs the asset allocation for that particular cell would determined by this setting.

It should be noted that not only is there is no "right answer" when determining an Objective Function, but they can be as complicated as you like.

Given our starting grid, the objective function and the basic functionality of the Static Model, the individual cells of the grid can be changed according to the following plan:

1) **Select a cell to vary.** It will be faster to select cells with a shorter planning horizon (since these cells will be used by simulations with a longer initial planning horizon) and with a higher withdrawal rate (since these will be more important when determining the Risk, however defined, of other simulations).
2) **Select a variety of settings for the cell.** For instance all possible combinations of allocations where the individual allocations are varied +/- 5% from the base setting.
3) **For each setting in (2), perform a full simulation**
   a. Select one-year returns for each asset class by the Monte Carlo method of the Static Model
   b. Calculate the remaining term and withdrawal rate at the end of this year. Note that the remaining term will not necessarily decline by with a 1:1 relationship to elapsed time: life expectancy will increase as each year is survived.
   c. Determine new asset allocations in accordance with the settings for the grid element determined in (b). Since annuities cannot be reduced, it will be necessary to adjust the Stock and Bond allocations from the cell setting if the portfolio allocation to annuities exceeds the desired allocation. It will also be noted that annuity pricing will vary as the client ages.
   d. Repeat from step (a) with this starting point until the end of the planning horizon for each run
   e. For the next run, start again from step (a)
   f. When enough runs have been performed to allow the objective function to score the setting, start again from (a) until all settings from (b) have been scored
4) **Determine the new settings for the cell.** This will be the setting that provides the highest value of the objective function
5) **Return to Step (1).** Repeat until a full cycle, covering each cell, has been completed with no indicated changes.

This will, obviously, require an enormous number of calculations to complete. So what? It's not a toy. There's very little in this world that is cheaper than microprocessor clock cycles, so dedicate a computer to the task, grinding out results seven days a week, twenty-four hours a day. If that's not fast enough, buy another computer and parcel out the work. If that's too expensive or too time-consuming, hire Hymas Investment Management to do it for you.

At the end of the process, you might find that Table A3 has been optimized into something like Table A5:

Table A5: Possible Final Asset Allocation Grid for Dynamic Model

Withdrawal Rate	Planning Horizon				
	1 Year	2 Years	……	29 Years	30 Years
1%	90% Stocks 10% Bonds 0% Annuity	95% Stocks 5% Bonds 0% Annuity	…	100% Stocks 0% Bonds 0% Annuity	100% Stocks 0% Bonds 0% Annuity
1.5%	85% Stocks 15% Bonds 0% Annuity	90% Stocks 10% Bonds 0% Annuity		95% Stocks 5% Bonds 0% Annuity	95% Stocks 5% Bonds 0% Annuity
….	…	…	…	…	…
9.5%	10% Stocks 30%% Bonds 60% Annuity	15% Stocks 20% Bonds 65% Annuity		20% Stocks 0% Bonds 80% Annuity	25% Stocks 0% Bonds 75% Annuity
10.0%	5% Stocks 30% Bonds 65% Annuity	10% Stocks 20% Bonds 70% Annuity		20% Stocks 0% Bonds 80% Annuity	25% Stocks 0% Bonds 75% Annuity

Note that the settings in each cell are for illustrative purposes only; there has been no attempt made to determine "correct" values.

Future Enhancement: Stochastic Demise

This essay is biased against bonds relative to annuities, due to its emphasis on a thirty-year survival rate (risk) and the thirty-year median portfolio value (reward). However, according to the mortality tables,[63] of 79,354 males who reach the age of 65, only 4,445 will reach the age of 95: a little less than 6%.

This does not mean we should not care about the portfolio's thirty-year survival rate, nor does it mean that we should disregard the projected thirty-year portfolio value: but it does mean that interim portfolio values are an important component of the reward component of a rational objective function.

The discussion in the section Annuities: Rates of Return makes the point perfectly clear: an annuity purchased at current rates by a 65-year-old male will underperform a 5.5% bond throughout the first 23 years of its existence – and his life expectancy is only 17 years. Perhaps we should determine just who it was who decided that a thirty-year planning horizon was appropriate for a retiree, and investigate him for possible links to the life insurance industry!

So, there are three possible solutions to this problem that I can see:

- Include interim portfolio values in the objective function, not simply the final portfolio value
- Include the minimum median portfolio value in the objective function
- Introduce another random element to the model, which terminates the simulation upon the demise of the retiree.

As an example of the second option I have performed a rough optimization, given the standard assumptions. Results are shown in Table A8, with the distribution graph of the last simulation shown in Chart A8.

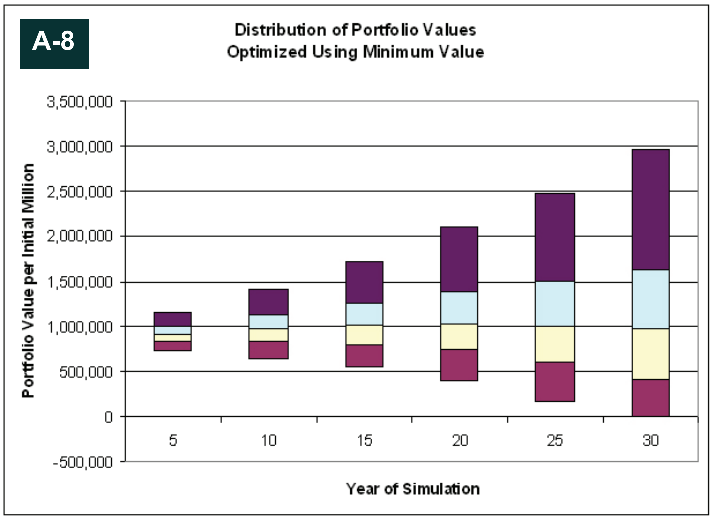
It will be noted that the 'Optimal' asset allocation is very different from the 20/50/30 optimal allocation determined in Table A7, which used end-value as its sole measure of reward.

Table A8: Successive Steps to an Optimal Portfolio Using Minimum Median Value [Settings not specified are as for the Initial Case, Table A4]

Stocks	Bonds	Annuities	Initial Withdrawal Rate	Ruin Rate	Minimum Median Value
45%	50%	5%	5.0%	17.6%	1,022,000
40%	50%	10%	5.0%	15.4%	968,000
40%	55%	5%	5.0%	16.2%	1012,000
35%	55%	10%	5.0%	13.1%	961,000
30%	60%	10%	5.0%	10.2%	963,000
25%	65%	10%	5.0%	8.0%	845,000
25%	60%	15%	5.0%	9.1%	888,000
30%	55%	15%	5.0%	9.6%	909,000

Notice that in the final simulation, the Minimum Median Portfolio Value is $909,000, despite the fact that with an Annuity Weighting of 15%, the initial portfolio value was $850,000 (the simulation recognizes only annuity income, setting annuity value to zero at all times).

It is interesting to note that for the last of these simulations, there is a clear pattern to the median portfolio value – it increases until year 20, then starts to decline as increased withdrawals start to overwhelm the bond coupon income. This is a very common pattern and is related to the fact that the "median portfolio" (if there is such a beast) is slightly cash-flow positive for the first fifteen years and is then increasingly cash flow negative.



[63] Social Security Online, *Period Life Table*, 2006, available on-line at http://www.ssa.gov/OACT/STATS/table4c6.html (accessed 2011-4-6)

Future Enhancement: Better Treatment of Bonds

As I have repeatedly complained in this essay, bonds are badly treated in this type of calculation. The requirement to specify a normally distributed annual return turns them into junior equities instead of allowing for their specific strengths and weaknesses.

A much better implementation would assume (like virtually all other fixed income mathematics) a flat yield curve, with the level of this curve being determined by some kind of reasonable distribution function. If, for instance, the normal yield is considered to be 5.5%, then a current rate of 4% might imply that the following year's rate will be lower 40% of the time and higher 60% of the time.

This enhancement then makes it possible to allow the user to select what kind of bond portfolio he wants:

- An initial bond with a specified term and coupon. The price each year is calculated from the prevailing interest rate and remaining term.
- A constant duration portfolio

A huge advantage of this, I believe, will be be to explode the fallacy that short-term bonds are less risky, in some kind of absolute sense, than long term bonds. I spend half my life pointing out[64] that fixed income management consists of a choice between security of principal and security of income: you can have one, but not both. A short-term bond portfolio exposes an investor's income to fluctuations in interest rates, while a long-term bond portfolio exposes his principal. If he has no intention of selling his principal then what he really needs is the security of income, which can taken to the logical extreme of a perpetual bond paying all his income needs.

Future Enhancement: Transaction Costs

As previously noted, I decided in the end to maintain a constant proportion between stocks and bonds in the market portfolio, after my initial attempt (taking market action on only one asset class every year) resulted in far too much drift from the idealized targets.

However, this involves transaction costs – not just the transaction costs represented by commission and the bid/ask spread, but also with taxes, particularly capital gains taxes on equity securities that have been held for a long time.

The tax advantages of holding a passive equity portfolio "forever" should be recognized in a future version of the spreadsheet.

Future Enhancement: Taxes

The spreadsheet, like every other implementation I have seen, does not differentiate in the tax treatment of dividends, capital gains and interest. (in addition to the capital gains effects mentioned in the above section).

Making this differentiation would add credibility to the spreadsheet's results.

Investment Conclusions

This has been a terribly long essay and I regret that I don't have any startling pearls of wisdom for those who have fought their way to this point.

However, in the course of preparing the materials for this essay, I became convinced that the major weakness of all these retirement calculators – my own included – is their static nature. Thirty years is simply too long a period to assess for planning purposes, due both to the chaotic nature of financial markets and the potential for demise or, more insidiously, a change in health of the retiree.

My advice is to remain as flexible as possible. Retirement plans should be updated annually, while eschewing the temptation to over-manage one's assets. Investors should focus on a 15-year plan (at the most) rather than a 30-year plan, while keeping a sharp eye not on the prospects for ruin, but for the prospects of large cuts in final results.

If, for instance, the first 15 years of retirement investment go badly, there is no need to continue with the same allocation for the next 15; and this should be recognized at year zero. In year 15 an annuity will be a lot cheaper than it is at year zero, and risk should be assessed with this in mind. In many cases, I suggest, an annuity purchase should be deferred, using the potential for annuity purchase as a safety net for one's retirement planning. After all, they grow a bit cheaper every day of your life! They also represent an irreversible decision – so plan to drop off your cheque on your way home from the doctor's office, not on the way there!

Cash is important. If at all possible, withdrawals from the portfolio should be funded by portfolio income; if there is a shortfall, consider shifting to higher yielding assets (without assuming too much risk, of course! It should be recognized that a higher cash yield will result in a lower expected capital gain). If that still does not solve the problem, an annuity should be considered.

And, by all means, don't take this or any other financial projection too seriously. They are useful as background, to allow you to play with the effects of different decisions, but they all rely on highly uncertain predictions of future events. Remember: I didn't predict a wave of revolts across the Middle East this year – and neither did anybody else.

[64] See, for instance, *Preferred Shares and GICs*, available on-line at http://www.himivest.com/media/PrefsAndGICs_090814A.pdf (accessed 2011-4-9)