Yield Calculation and FixedResets

In the appendix to the May, 2010, edition, we looked at the behaviour of FixedResets during their Slump Period from 2010-3-26 to 2010-4-29 and concluded that issues of this type are trading on the basis of Current Yield – that is, the current dividend divided by the price. There appears to be an adjustment to valuation based on the total expected capital loss.

This is despite the fact that this is a completely insane methodology. It ignores:

- The rate (total/time) of the expected capital loss should the issue be called (virtually a certainty for most extant FixedResets)
- The change in dividend should the issue not be called and the dividend reset for the ensuing five years to the defined spread about Canadas
- The proximity of the ex-Dividend Date[1]

Much the same thing is seen with PerpetualDiscounts, but this is less of an error; the actual yield will be equal to the Current Yield on every dividend payment date, provided that the holder is actually entitled to the next dividend. It would be unheard of, but not impossible, for this not to be the case – this would imply that the ex-Dividend date for dividend #2 occurs before the payment for dividend #1.

Another problem with yields is, surprisingly, the prevalence of electronic calculators and spreadsheets. As I noted in the April edition of this newsletter, fixed income yields are quoted in terms of conventions which have origins before electricity was even harnessed. The standard convention is that for an instrument paying its coupon N times per year,

- The yield is calculated using compounding periods equal to the period between the coupons. This results in a yield expressed per compounding period – normally six months for a bond and three months for a preferred share. There are N periods per year.
- The yield is quoted as an annual rate. The annual rate is determined by multiplying the periodic return by N.

Thus, for example, if we have a par bond paying $4 annually in equal semi-annual installments, the yield is 2% per half-year, which is multiplied by 2 to arrive at the quoted yield of 4%. If we were to calculate the Internal Rate of Return (IRR) for this bond – by convention, IRR is calculated with annual compounding – we arrive at:

$IRR = (1 + 0.02) * (1 + 0.02) - 1 = 4.04\%$

If the above instrument was a preferred share, paying its $4 in equal quarterly installments, we will still quote the yield as 4%, but:

$IRR = (1 + 0.01) * (1 + 0.01) * (1 + 0.01) * (1 + 0.01) - 1 = 4.06\%$

The differences may seem minor, but can lead to furious arguments[2] and may have important consequences when evaluating a trade; whenever an investor is told that such-and-such an investment yields X, he should always ensure he understands how the yield is expressed.[3] It is mathematically trivial to convert between conventions – but only if you know the convention which is to be converted!

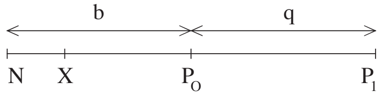
One problem with precise yield calculations is that there is no closed-form solution for determining the yield – the answer must be arrived at recursively,[4] using successively better approximations until the yield chosen results in the present value of the future payments being equal to the current price of the instrument. Unless one has an electronic program to perform this calculation, this can be a very laborious process; hence, investors have developed methods by which they can very quickly obtain an answer that is ‘close enough’ to the precise value.

In this appendix, I will develop two such approximation; one for use with PerpetualDiscounts and the other for use with preferred shares that are likely to be called. However, the latter derivation will work only on dividend dates – to my shame, I have as yet been unable to determine the closed-form approximation for instruments with a finite maturity not on the dividend date, although common sense suggests the adjustment will be identical to the adjustment for PerpetualDiscounts. I will keep plugging away at it – but anyone who puts me out of my misery and supplies a proof will have their work published with full credit.

I will also review the current market for FixedReset preferreds and see whether the conclusion arrived at in the May, 2010, edition still holds true.

Terminology and Reference

It is easiest to visualize the problem in terms of a time line.



In this diagram, we are at time N and wish to evaluate the yield of an instrument that has price P. At times P_0 and P_1 (and continuing into the future) the instrument will pay a dividend, d.

The time between N and P_0 is b days, subsequent payments are separated by an interval of q days.

1 See *Dividends and Ex-Dates*, Canadian Moneysaver September 2006, available on-line at http://www.himivest.com/media/moneysaver_060901.pdf

2 See, for example, *Research: Modified Duration*, and the comments thereto, on-line at http://www.prefblog.com/?p=864

3 The usual answer is “That’s what it says on my screen.”

4 You plug in your best guess and when it doesn’t work you curse. When your subsequent attempt doesn’t work, you recurse.

The ex-Dividend date for P_0 occurs after time N – by definition, or else the dividend at P_0 would not be earned. However, it should be noted that there could be a payment on the instrument not earned by the holder at any time prior to P_0 – the timeline is not drawn to scale.

We seek to determine the periodic yield, i, which will result in the Present Value of the future cash-flows, PV(), being equal to the current price, P.

The First Order Exponential Assumption will be used in developing the approximations in this essay. It states:[5]

For sufficiently small a, $(1 + a)^n$ is almost equal to $(1+an)$

Note that this approximation is used fairly often in the financial world. For example, the equation used when calculating yield with the Money Market convention[6] is:

$P = 100/(1 + id/365)$

Where i is the Money Market yield

P is the Price

d is the day count until maturity

And the value $(1 + id/365)$ is the first order approximation to $(1 + i)^t$ where t is expressed in years.

Precise Method of PerpetualDiscount Yield Calculation

$$P = PV(P_0) + PV(P_{1 \ldots n}) \qquad (1)$$

$$= \frac{1}{(1+i)^{b/q}} \bullet d + \frac{1}{(1+i)^{b/q}} \sum_{t=1}^{\infty} \frac{d}{(1+i)^t} \qquad (2)$$

$$= \frac{1}{(1+i)^{b/q}} \left(d + \sum_{t=1}^{\infty} \frac{d}{(1+i)^t} \right) \qquad (3)$$

$$= \frac{d}{(1+i)^{b/q}} \left(1 + \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} \right) \qquad (4)$$

LEMMA: Let $r = \frac{1}{(1+i)}$

$$\therefore \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} = \sum_{t=1}^{\infty} r^t$$

$$\text{Let } X = \sum_{t=1}^{\infty} r^t = r + r^2 + r^3 + \ldots$$

$$rX = r^2 + r^3 + \ldots$$

$$(1 - r) X = r - r^{\infty} = r$$

$$X = \frac{r}{1-r}$$

$$= \frac{1/(1+i)}{1-(^1/(1+i)}$$

$$= \frac{1}{1+i-1}$$

$$\sum_{t=1}^{\infty} \frac{1}{(1+i)^t} = \frac{1}{i}$$

$$P = \frac{d}{(1+i)^{b/q}} \left(1 + \frac{1}{i} \right) \qquad (5)$$

As discussed, this equation cannot be solved directly; recursive attempts must be made to obtain a valid answer until an acceptable degree of precision is achieved. For example, if one required three significant digits, one might actually compute four significant digits. Achieving further accuracy has very little point.

5 See Computational Infrastructure for Operations Research, *Second Order Exponential Approximation*, http://www.coin-or.org/CppAD/Doc/exp_2.htm (accessed 2010-6-11)

6 For excruciating detail regarding yield conventions, see the Investment Industry Association of Canada publication, Canadian Conventions in Fixed Income Markets, available on-line at http://www.iiac.ca/Upload/Canadian%20Conventions%20in%20FI%20Markets.pdf (accessed 2010-6-13)

Dividend Adjusted Price (DAP) Method of PerpetualDiscount Yield Calculation

We know that Current Yield is an accurate method of calculation PerpetualDiscount yields provided that the calculation is performed on a particular date relative to the issue's dividend cycle (the intuitive conclusion, that this date is the ex-Dividend date, is incorrect, as we shall see).

It seems reasonable to suppose that we can adjust the market price of the instrument to compensate for the issue's actual position in the dividend cycle and then proceed with the simple calculation. Thus, we will assert that we can write Equation (6), which we can see is precisely equivalent to equation (5) when a = b = q:

$$P - ad = \frac{d}{i} \qquad (6)$$

$$P = \frac{d}{i} + ad \qquad (7)$$

$$P = d\left(\frac{1}{i} + a\right) \qquad (8)$$

From Equations (5) & (8)

$$\frac{d}{(1+i)^{b/q}}\left(1 + \frac{1}{i}\right) = d\left(\frac{1}{i} + a\right) \qquad (9)$$

$$\frac{1+1/i}{(1+i)^{b/q}} = \frac{1}{i} + a \qquad (10)$$

$$a = \frac{1+1/i}{(1+i)^{b/q}} - \frac{1}{i} \qquad (11)$$

DAP Method: When is No Adjustment Required?

When there is no adjustment to the market price (that is, when a=0 in equation 6), the DAP method is equal to the Current Yield, and equation 11 shows that it will result in the calculation of the correct yield. So, under what conditions is the adjustment equal to zero?:

Let a = 0

From equation (11)

$$0 = \frac{1+1/i}{(1+i)^{b/q}} - \frac{1}{i}$$

$$\frac{1}{i} = \frac{1+1/i}{(1+i)^{b/q}}$$

$$1 = \frac{1+i}{(1+i)^{b/q}}$$

$$(1+i)^{b/q} = 1+i$$

$$\therefore b = q$$

Referring to the time line, we see this means that the Dividend Adjusted Price is equal to the actual price on dividend pay dates.

DAP Method: Can Current Yield Overestimate Yield?

Current Yield will overestimate yield when the adjustment increases the market price. It seems counter-intuitive that this should be case, but we can plug the adjustment into equation 11 and see whether this implies that specific conditions must hold, or whether this assumption will lead to a contradiction in the math:

Let $a < 0$

From equation (11)

$$0 > a = \frac{1+1/i}{(1+i)^{b/q}} - \frac{1}{i}$$

$$\frac{1}{i} > \frac{1+1/i}{(1+i)^{b/q}}$$

$$1 > \frac{1+i}{(1+i)^{b/q}}$$

$$(1+i)^{b/q} > 1+i$$

and since $i > 0$

$b > q$

Referring to the time line, we see that this is the case when the dividend on the next pay date will not be received; the first dividend is more than one quarter away.

DAP Method: Can Current Yield Underestimate Yield?

Most people know intuitively that this is the case, but as has been shown, intuition can be wrong! Current Yield will underestimate the actual yield under certain (quite common) circumstances, as may be seen when we examine the implications of $a > 0$:

Let $a > 0$

From equation (11)

$$0 < a = \frac{1+1/i}{(1+i)^{b/q}} - \frac{1}{i}$$

$$\frac{1}{i} < \frac{1+1/i}{(1+i)^{b/q}}$$

$$1 < \frac{1+i}{(1+i)^{b/q}}$$

$$(1+i)^{b/q} < 1+i$$

and since $i > 0$

$b < q$

Referring to the time line, we see that this refers to the period between the prior dividend's pay date and the next dividend's ex-date.

DAP Method: Approximating a Closed Form Solution

When examining equation (11) we remember the first order exponential approximation discussed in 'Terminology and Reference', above, and applying this to the exponential term in equation (11) results in

$$a \approx \frac{1+1/i}{1+ib/q} - \frac{1}{i} \qquad (12)$$

$$ai = \frac{i+1}{1+ib/q} - 1$$

$$= \frac{i+1-1-ib/q}{1+ib/q}$$

$$= \frac{i-ib/q}{1+ib/q}$$

$$a = \frac{1-b/q}{1+ib/q}$$

since ib/q << 1, we may make a further approximation and state:

$$a \approx 1 - b/q$$

$$a = (q - b)/q \qquad (13)$$

Substituting equation (13) into equation (8) gives

$$P = d\left(\frac{1}{i} + \frac{q-b}{q}\right)$$

$$= \frac{d}{i} + \frac{d(q-b)}{q}$$

$$P - \frac{d(q-b)}{q} = \frac{d}{i}$$

Define $P' = P - \dfrac{d(q-b)}{q}$ (14)

and then

$$i = \frac{d}{P'} \qquad (15)$$

Checking the DAP Approximation

Naturally, before we proclaim our success at deriving a closed form yield approximation for PerpetualDiscounts, we should check the results,; not merely to avoid embarrassment if any errors have crept in to the algebra, but to review the effect of the approximations we made. It is, after all, perfectly possible that the approximation loses considerable validity in the conditions in which we wish to use it – as I always stress, when we form empirical conclusions from data, we must bear in mind that these conclusions only apply in conditions similar to those of the data collected.

To many, the most surprising element of equation (14) is that the price adjustment can change sign. This behaviour will be most pronounced when there is a long period between the ex-Date and the pay-Date of any given dividend: in the interim the sign of the price adjustment will be negative (which means that the adjusted price, P', will be higher than the actual market price, P).

Accordingly, I reviewed data for 2010 and chose the longest interval I could find. The National Bank PerpetualDiscount issue NA.PR.L has paid two dividends so far this year, with characteristics[7] as shown in Table 1.

Table 1: NA.PR.L, Dividends Paid in 2010 to Date

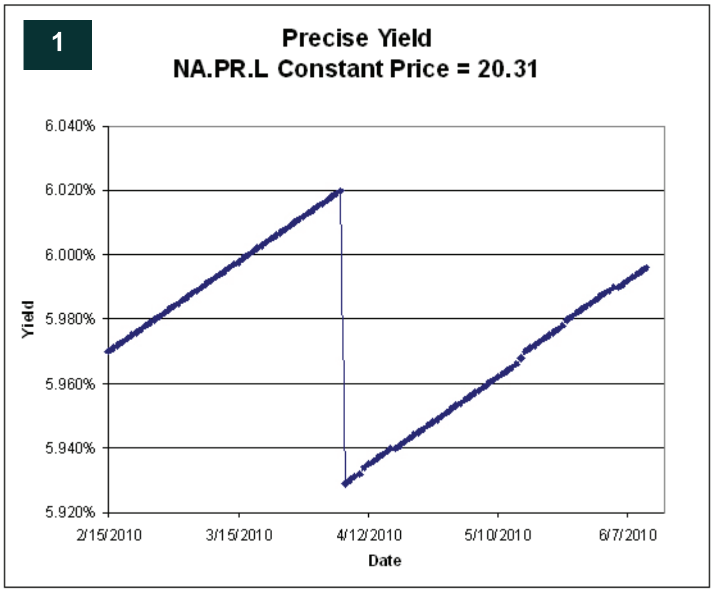
Identifier	Ex-Date	Record Date	Pay Date	Amount
Dividend 19	1/6/10	1/8/10	2/15/10	0.303125
Dividend 20	4/7/10	4/9/10	5/15/10	0.303125

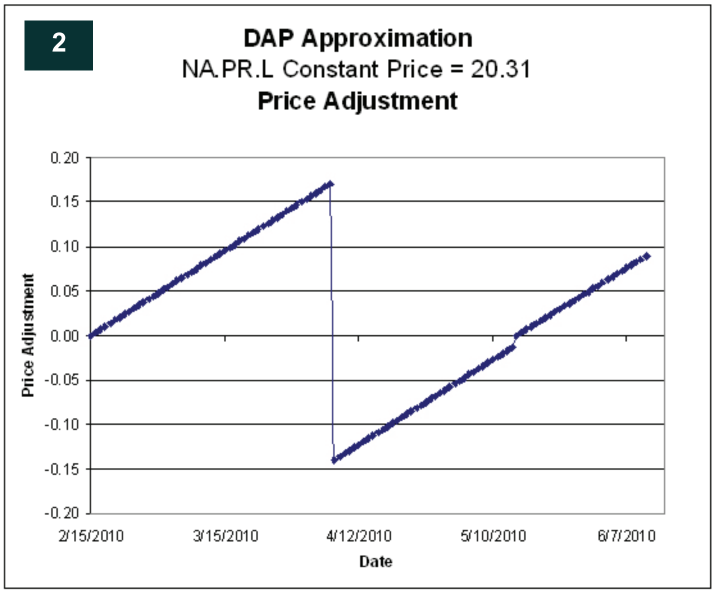
In order to highlight the effects of time, charts were prepared showing the calculated yield over time given a constant price of the issue of 20.31 (which was the closing bid on June 10). The sawtooth pattern that is so common in fixed income analysis is clearly visible in Chart 1, which shows the precise yield, calculated in accordance with Equation (5). Note that there is a little 'wobble' in the graph, an unfortunate byproduct of differing numbers of days in the next quarter; the parameter "q" changes from 89 to 92 on May 15. Perhaps it would have been better to set q to a constant figure based on four quarters in a 365-day year!

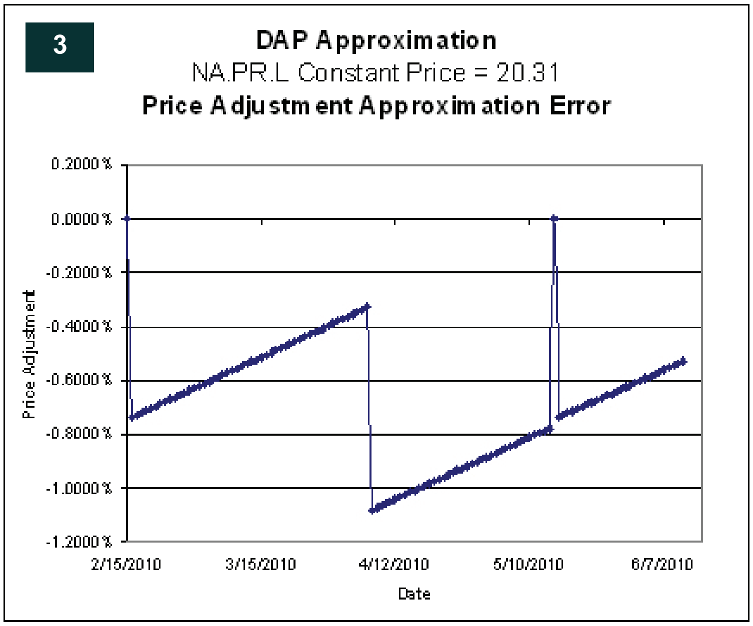
Chart 2 shows the price adjustment to be applied to the market price to account for the position in the dividend cycle on each day. This is the adjustment d(q-b)/q shown as the second term on the right-hand side of equation (14).

Finally, we recall that we were able to derive a mathematically correct expression for the required price adjustment in equation (11); this adjustment is not used in practice since it is dependent upon the value of i and therefore provides no relief from the torment of recursive calculations. Instead, it was approximated by equation (13). Chart 3 examines the difference between the two expressions (shown as a percentage of the true adjustment of equation (11)) an shows that the error introduced by the approximation used derive equation (12) is not significant.

7 See National Bank, *National Bank Declares Dividends*, Press Release, 2010-2-25, available on-line at http://www.nbc.ca/bnc/files/bncpdf/en/2/e_ri_9hGWXxeK16rd.pdf (accessed 2010-6-12) and National Bank, *National Bank Declares Dividends*, Press Release, 2009-12-3, available on-line at http://www.nbc.ca/bnc/files/bncpdf/en/2/e_ri_9hGWXxeK16rd.pdf (accessed 2010-6-12)







Similar Calculations for Instruments with Maturities

Preferred shares may have an explicit maturity (in the case of split shares, for example, when they are described as "hard maturities"); a maturity that is expected in all but the most dire circumstances (in the case of retractibles, the presence of the retraction privilege is presumed to trigger a call on or before a spedified date; these are called "soft maturities"); or a maturity that is currently presumed based on current market conditions (an issue with a high dividend may be presumed to be called when the company can – at least in theory – refinance at a cheaper rate; these are quite simply knows as "calls").

Equation (20) shows the formula by which the yield is calculated for these instruments on each dividend payment date; the algebra leading to the closed-form approximation of equation (23) will be familiar to those who have examined the calculations for PerpetualDiscounts.

$$P = \sum_{t=1}^{N} \frac{d}{(1+i)^t} + \frac{M}{(1+i)^N} \qquad (20)$$

LEMMA

$$X = \sum_{t=1}^{N} \frac{1}{r^t} = \frac{1}{r} + \frac{1}{r^2} + \ldots + \frac{1}{r^N}$$

$$rX = 1 + \frac{1}{r} + \ldots + \frac{1}{r^{N-1}}$$

$$(1-r)X = \frac{1}{r^N} - 1$$

$$X = \frac{1/r^N - 1}{(1-r)}$$

Substituting the Lemma into equation (20) with r = 1+i gives:

$$P = d\left(\frac{1/r^N - 1}{(1-r)}\right) + \frac{M}{(1+i)^N}$$

$$= d\left(\frac{1/(1+i)^N - 1}{1-1-i}\right) + \frac{M}{(1+i)^N}$$

$$= d\,\frac{1-1/(1+i)^N}{i} + \frac{M}{(1+i)^N}$$

$$= \frac{d}{i}\left(1-1/(1+i)^N\right) + \frac{M}{(1+i)^N}$$

$$P\,(1+i)^N = \frac{d}{i}\left((1+i)^N - 1\right) + M \qquad (21)$$

applying the first order exponential approximation to equation (21) provides

$$P(1+Ni) = \frac{d}{i} \bullet (1+Ni-1) + M$$

$$= \frac{d}{i}\,Ni + M$$

$$P + PNi = dN + M$$

$$PNi = dN + M - P$$

$$i = \frac{dN + M - P}{PN}$$

$$= \frac{dN}{PN} + \frac{M-P}{PN}$$

$$= \frac{d}{P} + \left(\frac{1}{N}\right)\frac{M-P}{P} \qquad (22)$$

$$= \frac{d+(M-P)/N}{P} \qquad (23)$$

Equation (23) is easy to understand: the true yield, i, is the current yield, d/P, plus an allowance for the amortization of the instrument's premium or discount, (M-P)/N.

To my chagrin, I must confess that I have not yet been able to generalize this equation to handle dates other than dividend pay dates. Common sense dictates that a price adjustment will have very similar – if not identical – form to that shown in equation (14); but I haven't been able to prove (or disprove!) this assertion as yet.

Should any readers care to send me a proof, I will be pleased to publish it[8], with or without acknowledgement, as desired.

[8] The first one, anyway. And I may not publish the proof exactly as supplied; some parts may be expanded, contracted or re-ordered as I see fit. But a proof is a proof, and will be acknowledged.

The Market Ignores All This

After examining the recent downdraft in FixedReset prices (March 26 to April 29) in the May edition of this newsletter, we concluded – at least as a hypothesis strongly supported by the data – that the FixedReset preferred market ignores all of this fancy math stuff and instead values individual issues based on two considerations:

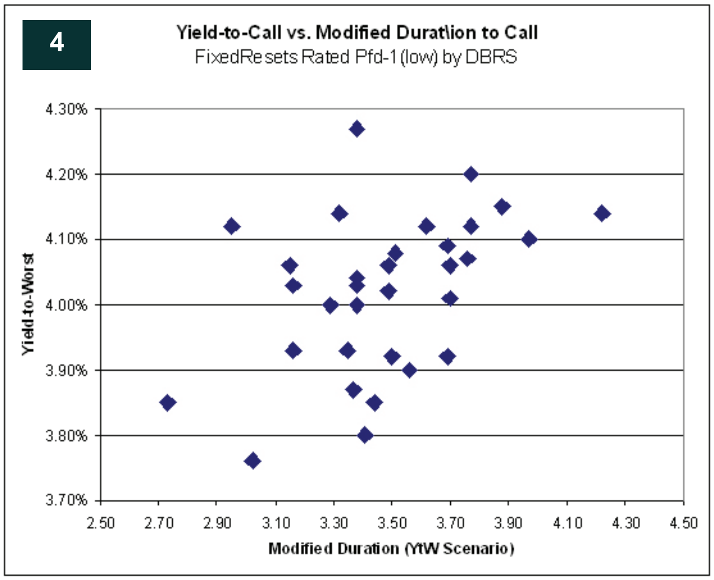
- Maximize Current Yield
- Minimize Expected Capital Loss

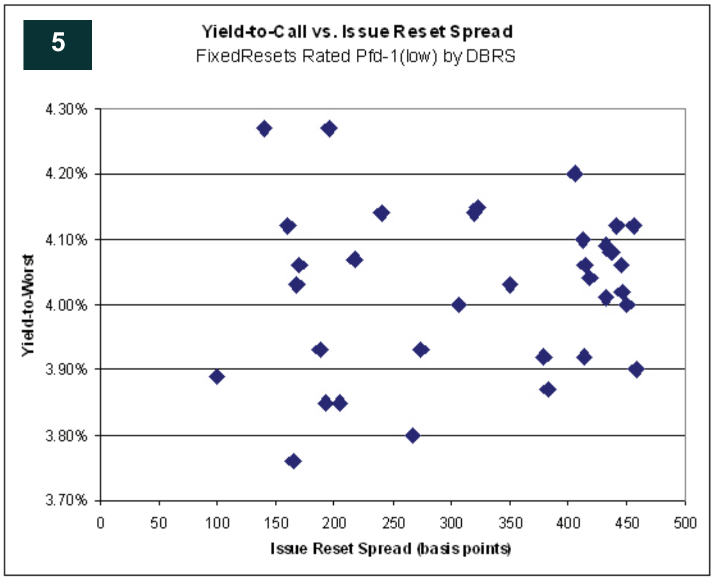
This simply does not make sense, since neither calculation makes an allowance for time – either the time that the current dividend will be in effect, or the time it will take for the expected capital loss (of an issue currently trading at a premium once it is called).

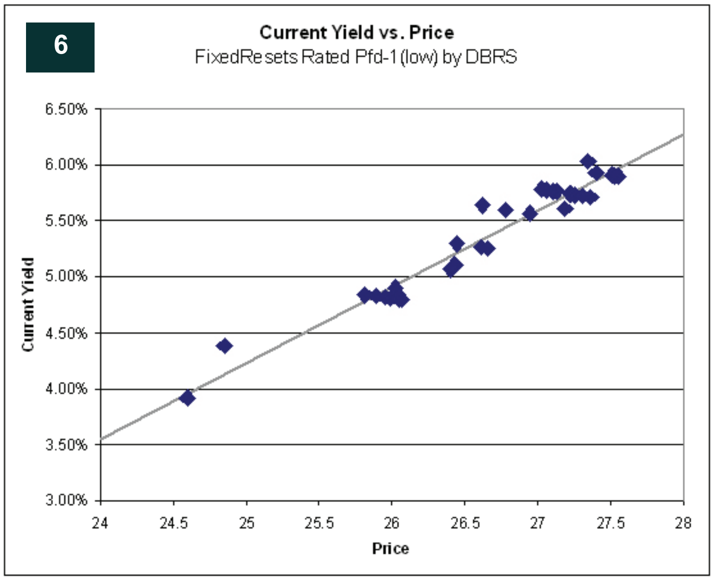
Normally, one would expect that the yield to call would be calculated and that the objective function would be stated in a manner more similar to the regular bond market – yield should increase as the term to maturity decreases. However, this does not appear to be the case, as shown from the June 11 data in Chart 4.

A more sophisticated adjustment – that may in some circumstances even become the dominant factor – is consideration of the certainty of maturity. Clearly, the higher the issue reset spread, the more likely it is that the issue will be called at the next call date; it would be entirely rational – and in accordance with the objective function stated above – for investors to accept lower yields in return for greater certainty that the issue will, in fact, be called. Again, though, this is not the case, as illustrated in Chart 5.

Instead, it appears that the faulty objective function defined in the first paragraph is still operable, as shown by the data in Chart 6. This insight has led me to hypothesize that there may, in fact, be a relationship between FixedReset and PerpetualDiscount yields that is based on Current Yield, rather than any more rational considerations.







The Relationship between FixedReset and PerpetualDiscount Yields

We may assume that investors assign a high value to the reset feature of FixedResets (although they really shouldn't, as discussed in the June, 2009, edition of this newsletter) and are prepared to accept a much lower yield in exchange for this feature.

However, their computation of the spread that they are giving up is hampered by their lack of computational ability: they compare Current Yields rather than Yields-to-Call.

We will test this hypothesis by collecting yield data (current yield and yield-to-worst) from the HIMIPref™ preferred share indices for the two types of issues from the inception of the structure. We will relate these four data series by spreads:

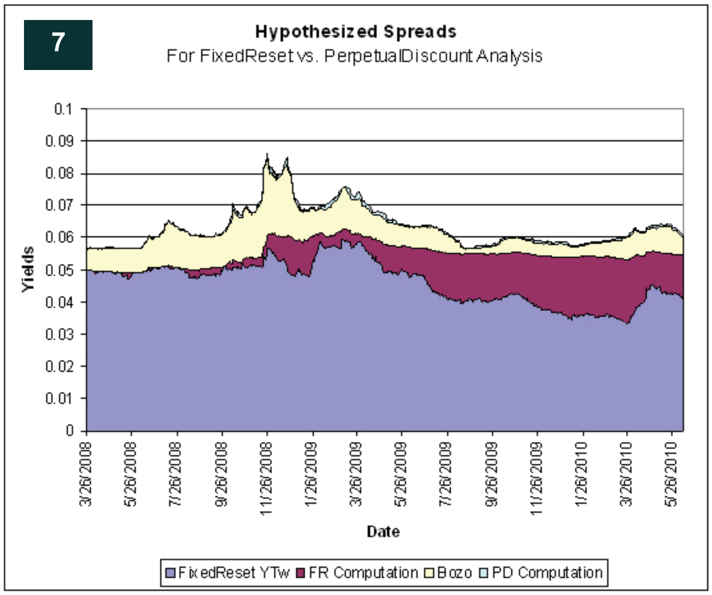
- **FixedReset Computation Spread:** the Current Yield less the YTW for FRs
- **Bozo Spread[9]:** the Current Yield on PerpetualDiscounts less the Current Yield on FixedResets
- **PerpetualDiscount Computation Spread:** The YTW less the Current Yield on PerpetualDiscounts.

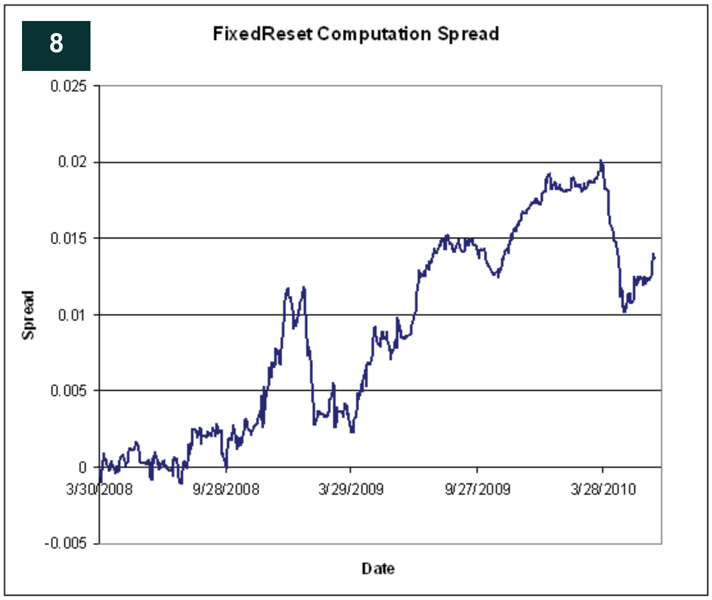
While it is difficult to imagine any of these spreads having any analytical value in a heterogeneous and efficient market, they may be of use in the market we actually have.

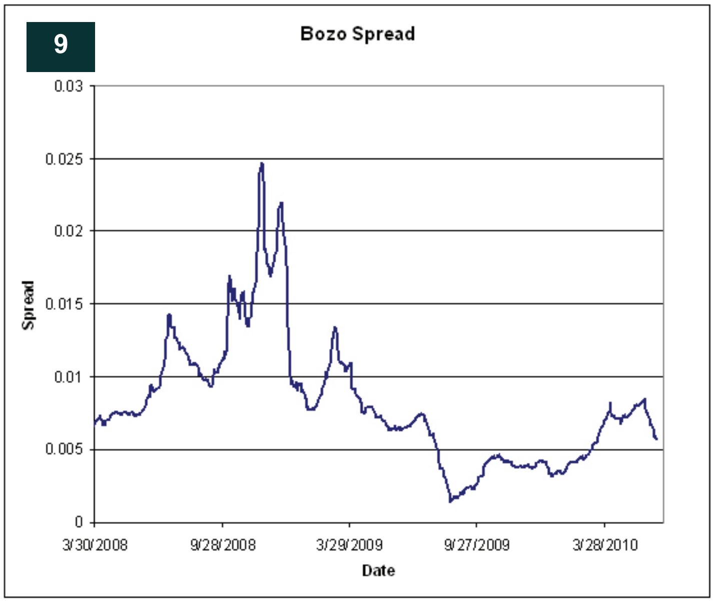
Charts 7–9 provide representations of these spreads. It is early days yet and I would be very hesitant to draw any firm conclusions from the data, but … the Bozo Spread has been remarkably stable for the past year, hasn't it? It will be most interesting to see if the Bozo Spread remains relatively constant as the FixedReset Computation Spread declines to zero.

I was tempted to supply an additional graph: the YTW of PerpetualDiscounts less the YTW of FixedResets. Such a relationship, however, contains a great deal of basis risk. FRs may be considered to be perpetual instruments: this will normally be the case when they trade below par and on occasion an issue with a relatively high current coupon and a relatively low Issue Reset Spread may legitimately trade above par while simultaneously being considered a perpetual instrument. Alternatively, issues with a high Issue Reset Spread will normally be considered to be likely candidates for a call at the next Exchange Date.

Hence, the yield relationship between FRs and PDs will vary, sometimes incorporating a term spread and at all times carrying some degree of uncertainty as to whether the call privilege will be exercised. At some time, I hope there will be a population of FRs with Issue Reset Spreads so low relative to market that it is universally recognized that they should be regarded as perpetuals, with perpetual credit risk – but such a population does not yet exist.







9 I'm not going to waste a good potential name for a meaningful spread on something as nonsensical as this!