Preferred Shares and Annuities[1]

Most investors have modest goals for their portfolios. They do not seek to dominate the world's iron-ore business; they do not seek to fund disaster relief in Africa for the next century; they simply wish to ensure that their retirements are as comfortable as they can be, given their saving and spending habits during their working lives.

Accordingly, the chief goal for most is preservation of income. Preservation of capital is definitely a nice thing to have; but given the choice between a comfortable retirement with a reasonable lifestyle and a miserly one devoted to increasing the inheritance of the beneficiaries, most will emphasize the former course, taking the view that their own comfort is of primary importance while making the kids entirely welcome to whatever happens to be left over.

Thus, there is an enormous literature available devoted to the concept of a Safe Withdrawal Rate: what percentage of a portfolio's value can be withdrawn per year without incurring too high a risk that the capital will expire before the retiree does?

Safe Withdrawal Rates (SWR)

The rule of thumb – that an investor can withdraw from his retirement portfolio at an initial rate of 4%, with this initial amount adjusted for inflation – contains some dangerous approximations. Generally speaking, there is no allowance for longevity risk aversion; additionally a rational retiree may well seek to increase spending early in retirement at the expense of late-retirement spending, on the grounds that there's a good chance he will not be around to care.[2] I suspect that many retirees take this course without the benefit of extensive mathematics proving that this behaviour is optimal!

Sharpe, Scott and Watson demonstrate[3] that the "4% rule" alluded to above is an inefficient implementation of a retirement financial strategy and that a "lock-box" strategy is efficient in the financial economist's sense: in their model the funding for the spending in each future year is assigned at the beginning of retirement and kept in a virtual lock-box. Every year, the appropriate lock-box is opened and all the assets held therein may be spent. While interesting in its rigorous approach to mapping investment strategies to optimal spending strategies, the concept is not practically useful: it requires an investor to know in advance the number of years of future spending that must be funded. If we knew that, many of our spending and saving decisions would be greatly simplified!

It is not enough to calculate the SWR based on a conservative long-term forecast of expected portfolio returns. Dr. Peter Ponzo (aka "gummy") has published[4] a brief tutorial and spreadsheet which shows the approximate effect on expected portfolio life when the standard deviation of returns is varied while the mean return remains constant.

Allied to the problem of volatility of returns is the effect of sequence of returns.

Sequence of Returns

Dr. Moshe Milevsky has examined this topic[5] and points out that while the sequence of returns makes no difference in the absence of cash flows, the effect of this perturbation can be quite profound.

Consider an investment portfolio that for the first five years of its existence achieves annual returns of -20.26%, 1%, 6%, 15% and 30%, for a total return of 27.63%, or 5% p.a. on an annualized basis. In the absence of cash flows, the order of these returns makes no difference; but if cash withdrawals of 5% of the initial value are made at the end of each year, this changes dramatically.

Table 1: Hypothetical Portfolio, Returns in Descending Order

Year	Starting Value	Portfolio Return	Year-end Value before cash flow	Cash flow	Final value
1	100,000	30%	130,000	-5,000	$125,000
2	125,000	15%	143,750	-5,000	$138,750
3	138,750	6%	147,075	-5,000	$142,075
4	142,075	1%	143,496	-5,000	$138,496
5	138,496	-20.26%	110,437	-5,000	$105,437

[1] This essay builds upon my post *Preferred Shares & Annuities*, available on-line at http://www.prefblog.com/?p=10109 and contains some text from a forthcoming book on preferred shares, due out (at the current rate of progress) sometime in the next century. Readers should be aware that Hymas Investment Management Inc. does not provide advice regarding insurance products or taxation; in discussing these matters I am stepping outside my area of professional competence. Readers should check facts with a fully qualified professional in these matters prior to taking investment action.

[2] M.A. Milevsky and H. Huang, *Spending Retirement on Planet Vulcan: The Impact of Longevity Risk Aversion on Optimal Withdrawal Rates*, available on-line at http://www.ifid.ca/pdf_workingpapers/Spending_Retirement_Vulcan_14MAR2010.pdf (accessed 2010-3-25)

[3] William F. Sharpe, Jason S. Scott & John G, Watson, *Efficient Retirement Financial Strategies*, Pension Research Council Working Paper Series, available on-line at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1005652 (accessed 2010-4-5).

[4] Peter Ponzo, *Safe Withdrawal Rates*, available on-line at http://www.financialwebring.org/gummystuff/SWR2.htm (accessed 2010-4-5)

[5] Moshe Milevsky, *Feast or Famine First?*, Research Magazine, December 2007, available on-line at http://www.ifid.ca/pdf_newsletters/PFA_2007DEC_FEAST.pdf

Changing the order in which the returns are experienced results in a dramatic difference in the ending value of the portfolio. As a result of his analysis, Dr. Milevsky concluded that – for a retiree simulated with reasonable parameters, and an average of 19 years of retirement – poor performance in the first seven years of the life of the portfolio had approximately double the effect of poor performance in the second seven years; which in turn had double the effect of the third seven year period (where "effect" was determined by the question of whether the initially chosen portfolio withdrawal schedule could be maintained).

Table 2: Hypothetical Portfolio, Returns in Ascending Order

Year	Starting Value	Portfolio Return	Year-end Value before cash flow	Cash flow	Final value
1	100,000	-20.26%	79,740	-5,000	$74,740
2	74,740	1%	75,488	-5,000	$70,488
3	70,488	6%	74,717	-5,000	$69,717
4	69,717	15%	80,175	-5,000	$75,175
5	75,175	30%	87,727	-5,000	$92,727

Dr. Milevsky has also pointed out that withdrawing the cash flow solely from the cash component of the portfolio is rather risky,[6] as it alters the asset allocation to a degree not contemplated in the forecast.

Thus it may be seen that a portfolio subject to negative cash flows will have ultimate results for the investor that are highly dependent upon the sequence of returns actually experienced. It is therefore necessary that an investor ensure that capital is preserved – and this is one of the great drawing cards for fixed-income.

Suppose the investor for whom we have examined the effect of sequence of returns invests his entire portfolio of $100,000 in perpetual government bonds with a yield of 5%. The analysis of his investment is summarized in Table 3.

In this manner, the investor has guaranteed himself – as well as it is possible to speak of guarantees in an uncertain world – his annual income of $5,000, which may be drawn each and every year forever. Market interest rates may go up or down; the capital value of his portfolio may change dramatically; but he can blithely cash his government cheques.

One could choose yearly changes in long-term government yields such that the total return of this perpetual bond matched that of the sample portfolio with ascending annual returns analyzed in Table 2 – and it wouldn't matter. The cash thrown off by this portfolio is constant, irrespective of market price. We will return to this point later in this essay, when discussing PerpetualDiscount preferred shares as annuity substitutes.

A fixed-income investor spending only the coupons is immune to sequence of return risk.[7] That is the whole point of fixed income.

Unfortunately, perpetual Government bonds are no longer available in Canada – there was an issue in 1936 (at 3%), but these were redeemed in 1996 after a twenty-year warning period.[8] The United Kingdom still has many such issues outstanding with a total face value of £2,885-million; the oldest of which carries a 2.5% coupon and was issued in 1853,[9] but this knowledge is of little value to Canadian investors!

Table 3: Hypothetical Portfolio, Invested in 5% Perpetual Government Bonds

Year	Starting Value	Income	Portfolio Return	Year-end Value before cash flow	Cash flow	Final value
1	100,000	+5,000	Who cares?	Who cares?	-5,000	Who cares?
2	Who cares?	+5,000	Who cares?	Who cares?	-5,000	Who cares?
3	Who cares?	+5,000	Who cares?	Who cares?	-5,000	Who cares?
4	Who cares?	+5,000	Who cares?	Who cares?	-5,000	Who cares?
5	Who cares?	+5,000	Who cares?	Who cares?	-5,000	Who cares?

6 Moshe Milevsky, *Can Buckets Bail-Out a Poor Sequence of Investment Returns?*, dated 2006-10-18, on-line at http://www.ifid.ca/pdf_newsletters/PFA_2006OCT_Buckets.pdf (accessed 2009-4-8)

7 Other risks remain, such as credit risk and inflation risk. Reinvestment risk will be present when the bond matures.

8 Serge Joyal, Commons Debate May 2, 1975, available online at http://www.sen.parl.gc.ca/sjoyal/e/debates/bonds_government_perpetual.html (accessed 2009-4-9)

9 UK Debt Management Office, *Annual Review 2007-08*, available on-line at http://www.dmo.gov.uk/documentview.aspx?docname=publications/annualreviews/gar0708.pdf&page=Annual_Review (accessed 2010-4-10)

Annuities

Milevsky noted[10] that – given reasonable assumptions of returns – an optimal and efficient solution to the SWR problem was to "pensionize" all assets, which means purchasing an annuity; this provides the greatest consumption of any plan, which he claims explains why most financial economists are strong advocates of this strategy.

A problem with this approach is that it does not address the major issue of desire for a legacy – zero value is assigned to any funds that are left over upon the investor's death. To a certain extent, this is a rational simplification of SWR theory, since those who must care the most about SWR are those who are closest to the edge: I do not imagine that Bill Gates spends much time agonizing over his retirement withdrawal rate!

Additionally, however, there is the problem of liquidity, which will be very hard to model. I am sure there are many readers who roll their eyes whenever I talk about liquidity as an important element in portfolio strategy and pricing but liquidity can have great consequences and many meanings. When retirement strategies are planned, for example, many people – including financial economists! – assume that there are only two states for an investor: dead and alive. This approximation makes the math a little easier but can cause an overemphasis on annuities, which are not liquid at all.

Dead and alive are not the only states and retirees do not, as a rule, drop dead immediately after drawing the losing ticket in the mortality pool. It is not unusual for a senior to be told of a degenerative disease; of impending blindness; or for other medical problems to be diagnosed that drastically alter his planning horizon. In these circumstances, the ability to shift consumption into the period in which it can be best enjoyed[11] has immense value, and this change in consumption timing is not possible with an annuity.

However, given the theoretical recommendation of annuities, it is prudent to understand their mechanics, value and weaknesses so that they may be given consideration as an element – perhaps even the dominant element – in a retirement strategy. Annuities do, indeed, have many good points.

Types of Annuities

An annuity is[12] a contract whereby *in exchange for a single payment, a life insurance company provides the annuitant with regular income payments, generally for the rest of the annuitant's life*.

The benefits of an annuity are best explained by the concept of a tontine annuity: such a scheme has a rich and not altogether sterling history.[13] *A tontine is an investment scheme through which shareholders derive some form of profit or benefit while they are living, but the value of each share devolves to the other participants and not the shareholder's heirs on the death of each shareholder. The tontine is usually brought to an end through a dissolution and distribution of assets to the living shareholders when the number of shareholders reaches an agreed small number.*

Annuities are no longer provided by tontine groups – although I'm sure that there are still a few extant as private contracts – but by insurance companies. Those seeking to purchase an annuity need no longer join a large group of people with similar characteristics and investment goals; instead, it is the insurance company that acts as counterparty and for each contract forms a 'virtual tontine' providing similar benefits, while attempting to make a profit. It's a big business: Sun Life reported[14] that as of year-end 2008, liabilities for individual and group annuities comprised $33.5-billion, or 41% of the total $81.4-billion in actuarial and other policy liabilities.

This essay will concern itself only with the simplest form of an annuity: prescribed single life with no guarantee.

There are, of course, a great number of insurance products offered besides such plain-vanilla annuities, but it seems fair to say that these are simply various combinations of simpler elements. An annuity with a guaranteed payment period, for example, can be replicated with a straight annuity and life insurance. As David O'Leary of Morningstar observed[15] when Manulife introduced its "Income Plus" product[16]: *It simply combines some of the benefits of an annuity with those of a segregated fund*.

This is not a forum for the discussion of the various products offered by insurance companies and it is not a topic on which I pretend a high degree of familiarity. However, my experience as a bond investor leads me to advise a very jaundiced eye be used when examining highly structured products: each bell and every whistle added to the plain vanilla structure may be expected to increase the profitability of the product for the issuer, while the complexity and unique nature of every package makes comparisons difficult – which is, in my experience, the whole point.

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[10] In *Spending Retirement on Planet Vulcan: The Impact of Longevity Risk Aversion on Optimal Withdrawal Rates*, supra

[11] In mathematical terms, changing the objective function mid-way through the experiment. Financial economists do not have to deal with the potential for such a change. I suggest that the impact of such a possibility dwarfs the mathematical inefficiencies in practitioners' SWR models.

[12] CANNEX Financial Exchanges Ltd, *Products: Annuities*, available on-line at http://www.cannex.com/canada/english/products_antc.htm (accessed 2010-4-6)

[13] Kent McKeever, *A Short History of Tontines*, available on-line at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1340062 (accessed 2010-4-6)

[14] Sun Life Financial, *Annual Report 2008*, available on-line at http://www.sunlife.com/static/global/files/Annual%20reports/2008/pa_e_2008AR_annual_report.pdf

[15] David O'Leary, *Our thoughts on Manulife's new guaranteed minimum withdrawal product*, Morningstar, 2006-11-15, available on-line at http://www.morningstar.ca/globalhome/industry/news.asp?articleid=ArticleID1114200613231 (accessed 2010-3-28)

[16] Manulife, *Income Plus*, available on-line at http://manulifegifselect.ca/incomeplus/introduction_video (accessed 2010-3-28)

Thus, the elements of the plain vanilla annuity discussed in this essay are:

Prescribed: Prescribed annuities are generally preferred by investors and advisors as they are taxed on a preferential basis,[17] but this preference is accompanied by the requirement that they be purchased with non-registered funds, among other conditions[18]. The taxable portion of a prescribed annuity is:

$$\text{Taxable Income} = \text{Annual payment} - \text{Single premium/Life Expectancy}$$

So, for example, if a client pays $100,000 for an annuity paying $8,460 annually and has a life expectancy of 13.76 years, the taxable income is calculated as

$$\text{Annual Taxable Income} = 8{,}460 - 100{,}000/13.76 = \$1{,}192$$

The remaining portion of the annual payments, 8,460 – 1,192 = 7,268, is considered (quite rightly!) to be return of capital and is not taxed – even if the annuitant should live longer than expected and hence have more capital returned than he originally deposited. A nice little bonus with this treatment is that the government uses the 1971 mortality tables[19] for this calculation; the slightly shorter life expectancy increases the return-of-capital amount to above what might be expected today, decreasing the taxable amount.

Single Life: A single life annuity contract provides payments to a single annuitant; when the annuitant dies, the contract is completed. The alternative is "joint and survivor life", which is purchased with two initial beneficiaries and continues until both have died.[20] This latter type can be closely replicated with a single life annuity accompanied by life insurance; if the annuitant is the first of the couple to die, the insurance proceeds could be used by the survivor to purchase a new single life annuity. However, even rough equivalencies of this nature will be very dependent upon the pricing of annuities by the insurance companies; it is my understanding that joint life annuities are quoted[21] based solely on the age of the primary annuitant. I find this difficult to believe, but if true, anticipate becoming highly attractive to young ladies in my golden years!

No Guarantee: The plain vanilla annuities discussed in this essay have no guarantee. Guaranteed annuities are those which will pay the agreed income for a minimum length of time; if the annuitant should die before the guarantee period expires, the annuity income will be paid to his beneficiaries until the period expires.[22]

Relative Pricing of Annuities

Moshe Milevsky and CANNEX have developed[23] the Implied Longevity Yield Index.[24] This index calculates the rate of return that must be achieved by a portfolio to justify delaying the decision to invest in a life annuity: assume that an investor has a certain amount of money that will, eventually, be invested in an annuity. He wishes to determine whether it is better to annuitize immediately with a ten-year guarantee, or to manage his money on his own for ten years, at which time he will use the portfolio value (after withdrawals equal to what he would have received with immediate annuitization) to purchase a non-guaranteed annuity.

If he assumes that the pricing of annuities will remain constant, what rate of return on his portfolio must he achieve for ten years in order to break even?

Or, in Dr. Milevsky's language, *We compute the internal rate of return that an x-year old (born in year y) would have to earn on the non-annuitized portfolio over the next τ years in order to replicate the income payout from the annuity and still be able to acquire the same income pattern at age $x + \tau$; assuming current pricing remains unchanged*.

Obviously, any combination of initial age, x, and waiting period, τ, may be used to calculate the implied longevity yield, but Dr. Milevsky chose an initial age of 65 years and a ten year waiting period to be the standard for which data are published.[25]

The equation used to compute ILY is not a standard Internal Rate of Return calculation. Instead, it is derived as:

$$\alpha_2 - (\alpha_1 - (1/\delta))e^{\delta\tau} - 1/\delta = 0 \quad (1)$$

where:

α_1 = 100000/Immediate Annual Payment
α_2 = 100000/Delayed Annual Payment
δ is the Implied Longevity Yield (ILY), for which we solve
e is the base of natural logarithms, 2.718…
τ is the delay period, in years

There are disadvantages to this specification. A theoretical objection is that real-world annuity payments are not received continuously, they are received monthly. A more practical objection to the published format is that the ILY is expressed as a continuously compounded yield, whereas street conventions are for quarterly compounding (most preferred shares), semi-annual compounding (bonds) and annual compounding (IRR calculations, which can result in disputes over yield and related calculations[26]). Fortunately, conversions between yield conventions are easy – provided one always knows the convention used for any quoted number! In this essay, these differences will be ignored.

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17 Standard Life, *Taxation of Non-Registered Enriched (or Impaired) Annuities*, April 2007, available on-line at http://www.advisors.standardlife.ca/en/pdf/reference/taxing/6139.pdf (accessed 2010-3-26)

18 See Manulife, *Taxation of Non-Registered Prescribed Annuity Contracts*, October, 2009, available on-line at https://hermes.manulife.com/CANADA/repsrcfm-dir.nsf/Public/TaxTopicTaxationofNonRegisteredPrescribedAnnuityContracts/$File/tepg_taxtopic_TxPrAn.pdf

19 Standard Life, supra

20 Manulife, *What Types of Annuities are Available?*, available on-line at http://www.manulife.ca/canada/investments.nsf/Public/guaranteed_annuities_types (accessed 2010-4-7)

21 See CB Online, *Annuity Rates – Joint Life 25-Year Guarantee*, available on-line at http://www.canadianbusiness.com/my_money/rates/annuity_joint_life_25-year_guarantee/index.jsp (accessed 2010-4-7)

22 Organization for Economic Co-operation and Development, *Glossary of Statistical Terms: Guaranteed Annuity*, available on-line at http://stats.oecd.org/glossary/detail.asp?ID=5242 (accessed 2010-4-7)

23 Moshe Milevsky, *A Note on Developing an Index for Life Annuities*, available on-line at http://www.ifid.ca/pdf_workingpapers/WP_2004Sept15.pdf (accessed 2010-3-25)

24 For values of this index, see IFID, Payout Annuity Index, on-line at http://www.ifid.ca/payout.htm (accessed 2010-4-8)

25 See IFID, *Payout Annuity Index*, available on-line at http://ifid.ca/payout.htm (accessed 2010-4-7)

26 E.g., James Hymas, *Research: Modified Duration*, post on PrefBlog, on-line at http://www.prefblog.com/?p=864

The IFID data for March 3, 2010 is:

- a 65-year old pays $100,000 for an annuity paying $658.70 monthly with a ten-year guarantee
- A 75-year old pays $100,000 for an annuity paying $904.35 monthly with no guarantee

In the IFID calculation of the ILY for March 3, 2010, the values are:

α_1 = 100000/Immediate Annuity Payment = 100000/(12*658.70) = 12.6512
α_2 = 100000/Delayed Annuity Payment = 100000/(12*904.35) = 9.2147
τ = 10 [years]

which results in

δ = Implied Longevity Yield = 5.91%

A computation of the Internal Rate of Return with values:

-100,000 on 2010-3-3

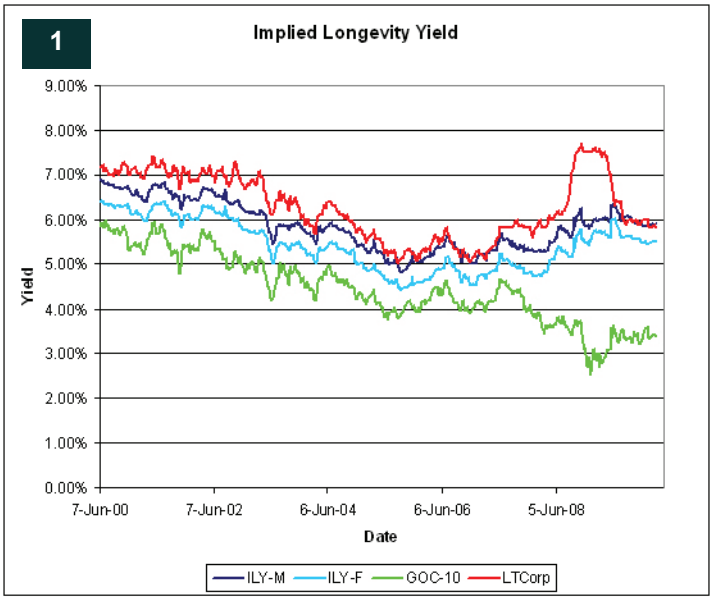
+658.70 on the third day of each month, commencing 2010-4-3 and ending 2020-3-3 (inclusive)

+72,836.84 on 2010-3-3 (calculated as 100000*658.7/904.35, the amount required for a 75-year old to purchase an annuity paying 658.70 monthly).

results in an IRR of 6.06% (which is 5.97% when expressed as a bond-equivalent yield with semi-annual compounding, and 5.88% when expressed as a continuously compounded yield).

In the original paper dated 2004, Dr. Milevsky stated *Figure #2 illustrates the evolution of the ILY values, compared to the yield on a 10-year Government of Canada Bond. We have chosen the yield on this particular fixed-income instrument given its centrality in many of the insurance companies' pricing algorithms, as well as the fact that it is a reasonable investment alternative to purchasing a life annuity at age 65.*

However, as shown in Chart 1, the credit crunch has caused this close relationship to break down – it caused a lot of relationships to break down! It appears that the ILY-65 is more closely related to the yield on long corporate bonds.



A correlation to corporates rather than Canadas is reasonable when considered in conjunction with Sun Life's asset allocation as of 4Q2009,[27] which is shown in Charts 2 and 3. It should be apparent that Sun Life's investment results will be far more dependent upon the corporate bond market than on government bonds.

Chart 2: Sun Life Invested Assets
Dec 31, 2009

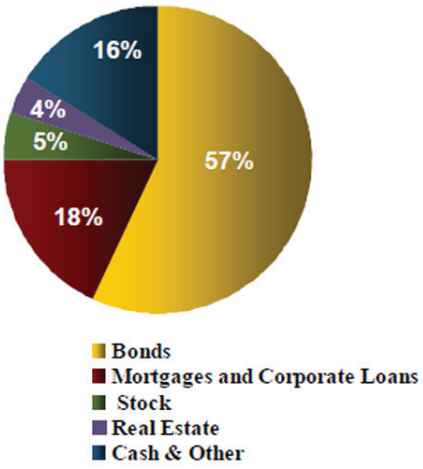
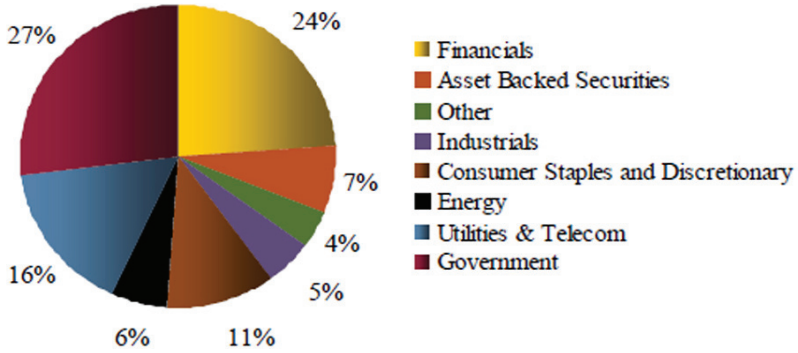


Chart 3: Sun Life Bond Sector Breakdown



[27] Sun Life, 4Q09 Investor Presentation, available on-line at http://www.sunlife.com/static/global/files/Quarterly%20reports/pa_e_Q409_presentation.pdf (accessed 2010-4-7)

Absolute Pricing of Annuities: the Insurers' Perspective

In the above section, we used Milevsky's concept of Implied Longevity Yield to gain some degree of understanding about the relative pricing of annuities: given one annuity price, it is relatively straightforward to compute annuity prices for other ages.

However, that calculation did not provide any insight as to the pricing of the first annuity. To determine this price, we must use mortality tables to estimate how long the insurer's commitment to pay a regular monthly income will be in force.

As a first step, we will take the single-life annuity payout for a 75-year-old man from the IFID data[28] and plot these data (as an annual percentage of the cost of the annuity) with a comparison to the long corporate rate in Chart 4. At a glance we can see these values are closely related and regression analysis, reported in Table 4, confirms this impression. The "goodness of fit" to the data (the "R-squared" regression result) is substantially in excess of the value Milevsky reported in his original paper[29] although it must be borne in mind that Milevsky was regressing changes in yield, not absolute yields, using Canada bonds rather than corporates and with a much smaller sample of data.

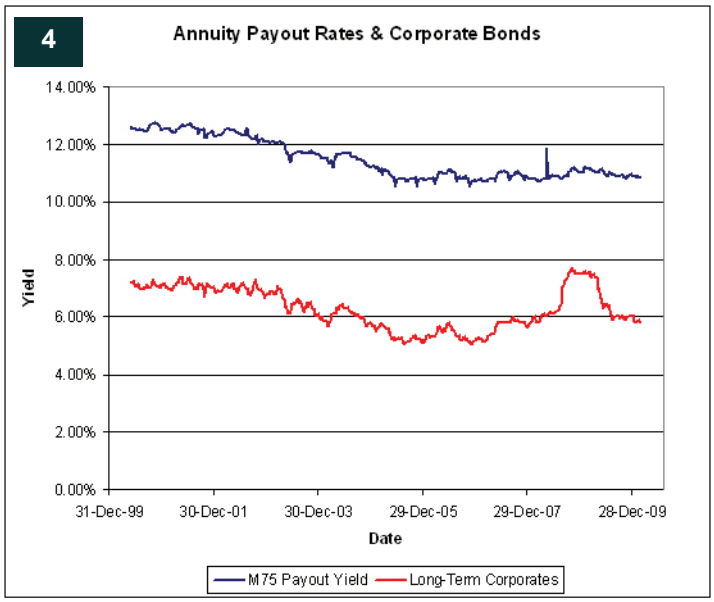


Table 4: Results of Regression
Annuity Payout = (slope) * Long Corporate Yield + (intercept)

Value	All Data to 2010-3-3	Data Prior to 2008-6-30 only
R-squared	0.54	0.92
Intercept (corporate yield = 0)	7.17%	5.71%
Slope	0.68	0.95

As may be seen, the correlation was quite good until the third quarter of 2008, when the Lehman bankruptcy caused a lot of correlations to vary from historical norms! Chart 4 indicates that the insurers could have increased annuity payouts dramatically during the depths of the crisis – but at the time they were focusing more on survival than market share and may have felt that the extraordinarily high level of long corporate yields was a transient effect.

Of more interest is the existence of the intercept, with the pre-Lehman correlation indicating that, as a rough rule of thumb, the annuity payout ratio (prescribed, single life, 75-year old man) will be about 5.75% above the yield on Long Corporates. The insurers do not pay this extra yield out of the kindness of their hearts, or because they are able to achieve enormous returns on the (dwindling) invested capital: this is the effect of the return of the annuitants' collective capital at a rate which will reflect, to some degree, the annuitants' life expectancy.

Our next step will be to evaluate that 5.75% number, which is based on the entire invested amount. Should a prospective annuitant consider it a "fair" rate for giving up the entire purchase price on day one?

Annuities as an Investment

For the ensuing computations, I will use the Social Security Life Table (SSLT),[30] as of 2005, published by the US government; another potential source of mortality data is available from Standard Life.[31]

According to the SSLT a seventy-five year old man has a life expectancy of 10.26 years, and according to the IFID ILY data, an investment of $100,000 would have provided a life annuity of $904.35 monthly as of March 3, 2010. If we calculate the Internal Rate of Return based on the initial payment 2010-3-3, followed by 123 monthly receipts in the time following, we arrive at a figure of 2.12% – which is a pretty poor return on investment. In order to achieve the 5.91% internal rate of return suggested by the ILY on 2010-3-3, the investor would have to receive monthly payments until May, 2023 – in other words, live for over thirteen years, significantly in excess of expectations.

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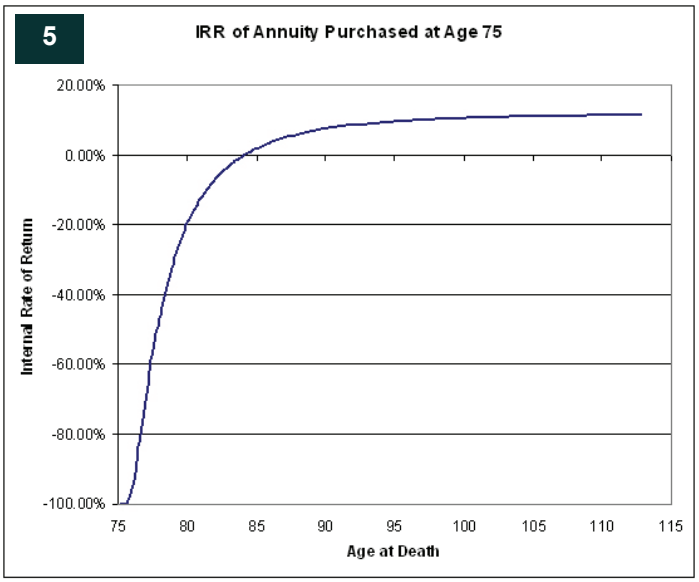
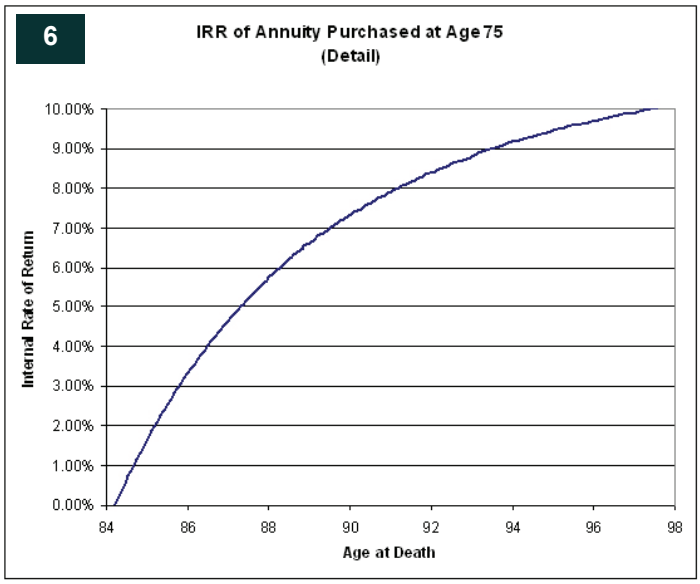
[28] IFID, *Payout Annuity Index*, supra

[29] Moshe Milevsky, *A Note on Developing an Index for Life Annuities*, supra

[30] Available on-line at http://www.ssa.gov/OACT/STATS/table4c6.html

[31] Available on-line at http://www.advisors.standardlife.ca/en/pdf/reference/4984.pdf





Charts 5 and 6 illustrate the internal rate of return that will be realized by the investor given various ages at death. As I have noted elsewhere[32] the yield that will be realized will reach a limit, no matter how long the annuitant lives: in this particular case, the limit resultant from the monthly payment of $904.35 from an investment of $100,000 is a periodic return of 0.904% per month, or 10.85% expressed as a monthly compounded yield, which is an IRR (yearly compounding) of 11.41%.

Given that an investor must live in excess of the overall expectancy in order to realize a good yield, one might suppose that potential annuity buyers will engage in a certain amount of self-selection prior to buying; and it is indeed the case that adverse selection is a factor that the insurance companies must account for when preparing their actuarial estimates.

In order to get a better understanding of this adverse selection, we can examine a pool of annuities formed from the general population and use the SSLT to determine the internal rate of return that will be required by an insurer in order to break even on the deal. Table 5 shows an extract of this calculation.

In Table 5, we begin with a pool of 60,102 male annuitants[33], all aged 75, who pay $100,000 each for annuities paying $904.35 monthly. The insurance company receives total premia of just over $6-billion. At the end of the first year, 57,522 annuitants are still living, so the company pays out a little over $624-million and the process continues until the last survivor dies after having received thirty-five years of payments.

We can make two observations from Table 5: first, the total dollars paid out is only slightly larger than the number of dollars paid in (the difference is less than $356-million, about 6% more than the insurer received in premia). Secondly, and related to the first observation, the insurer only has to realize a rate of return of 0.80% in order to break even.

Unfortunately for the insurer, there is no guarantee that the mortality of the annuitants will be equal to that experienced by the population at large. Suppose that, of the initial pool of 60,102 potential customers, the 2,580 who are expected to die in the first year have certain knowledge of that fact and therefore decline the invitation to purchase the annuity. We can then repeat the calculation, changing the first row to reflect 57,522 annuitants paying a total of $5,752,200,000 in premia, with all other rows unchanged. When the IRR is recalculated, we find that the insurer must now earn 1.43% on its invested money in order to break even – adverse selection has made it harder for the insurer to earn a profit.

But what if all those who will die within two years know that for a certainty and don't buy? Three years? The results of such speculations are summarized in Table 6.

Table 5: Calculation of Insurers' Internal Rate of Return on Single Life Annuity with No Guarantee, Using Standard Mortality Tables.

Year	Lives	Cash Flow	Discounting Factor (Yield = 0.80%)	Present Value
0	60,102	+6,010,200,000	1	+6,010,200,000
1	57,522	-624,240,248	0.992044	-619,274,041
2	54,809	-594,798,230	0.984152	-585,371,918
3	51,974	-564,032,243	0.976323	-550,677,404
4	49,029	-532,072,514	0.968555	-515,341,667
5	45,986	-499,049,269	0.960850	-479,511,432
…	…	…	…	…
33	4	-43,409	0.768292	-33,351
34	2	-21,704	0.762180	-16,543
35	1	-10,852	0.756117	-8,206
36	0	0		0
Total		**-355,798,190**		**0**

[32] James Hymas, *Perpetual Misperceptions*, Canadian Moneysaver September 2007, available on-line at http://www.himivest.com/media/moneysaver_0709.pdf

[33] The number 60,102 is used for convenience, as the SSLT shows this as being the number of 75-year-olds resulting from the live birth of 100,000 males.

As discussed in the section "Relative Pricing of Annuities", it appears that on 2010-3-3, insurers were using an internal rate of return of 5.91% to come up with relative prices for annuities. The results shown in Table 6 imply that – in the absence of any other factors, such as commissions paid to salesmen, other expenses and profit – this rate of return is consistent with a significant adverse selection experience. The price of the annuity becomes self consistent only if nearly half of the potential buyers of the annuity decide not to buy on the grounds that they have certain knowledge that they will die prior to their expected time.

Table 6 also reports the Macaulay Duration of the cash-flows derived in the course of the computation – it is interesting to note that as the period of foreknowledge increases this figure first declines (as the effect of increasing required yield increases) and then increases (as the effect of the longer average term of payments to each annuitant increases). Approximate data for the Long and Mid-term Corporates Index[34] are reported in Table 7 together with data on PerpetualDiscount preferred shares; it appears that to match a Macaulay duration of 6.7 years (the first step in immunization[35]) then a portfolio comprised of 88% mid-term and 12% long term corporates will be required – but such a portfolio will have a yield of only about 4.74%. It's a cruel world for fixed income portfolio managers!

Table 6: Effect of Adverse Selection due to Foreknowledge of Death On Required Rate of Return for the Insurer

Years of Potential Annuitant Foreknowledge	Annuities Purchased (from population of 60,102 possible)	Internal Rate of Return for Insurer break-even	Macaulay Duration
0	60,102	0.80%	7.1
1	57,522	1.43%	6.9
2	54,809	2.07%	6.8
3	51,974	2.70%	6.7
4	49,029	3.32%	6.7
5	45,986	3.92%	6.7
6	42,849	4.49%	6.7
7	39,626	5.04%	6.7
8	36,336	5.55%	6.8
9	33,006	6.03%	6.9

Table 7: Approximate Corporate Bond Index Data

Value	Mid-Term (5–10 year) Corporates	Long-Term (10+ year) Corporates	PerpetualDiscount Preferreds
Yield	4.6%	5.75%	6.23% (Dividend) or 8.72% (Interest Equivalent)
Modified Duration	5.9	11.5	16.1 or 11.5
Macaulay Duration	6.0	11.8	17.1 or 12.5

These calculations were performed for illustrative purposes; there are many other factors that need to be taken into account when pricing annuities. The actuarial computations are complex, require occasional revision and do not come with a guarantee: Sun Life disclosed in 2008[36] *For annuities for which lower mortality would be financially adverse to the Company, a 2% decrease in the mortality assumption would decrease net income by about $80 million…*

Risks in estimating mortality are a major factor for insurers and, such are the wonders of modern financial engineering, there are derivatives available whereby these risks can be hedged. According to JPMorgan[37], *LifeMetrics is a toolkit for measuring and managing longevity and mortality risk, designed by J.P. Morgan for pension plans, sponsors, insurers, reinsurers and investors. LifeMetrics enables these risks to be measured in a standardized manner, aggregated across different risk sources and transferred to other parties. It also provides a means to evaluate the effectiveness of longevity/mortality hedging strategies and the size of residual risk … our goals in launching LifeMetrics are to … Build a liquid market for longevity derivatives.*

It is also interesting to note Aronoff and Milevsky's demonstration[38] that the Implied Longevity Yield increases with age, as shown in Table 8. This would be consistent with the effect on the insurer of adverse selection increasing with the age of the potential annuitant; as one ages one becomes more aware of one's health relative to the norm for that age – not necessarily in terms of years, but in terms of peer rank.

Table 8: Variation of Implied Longevity Yield by Age, Gender and Wait-Time.

Age/Gender	5 Years Wait	10 Years Wait	15 Years Wait
62M	6.8%	6.8%	6.8%
67M	7.3%	7.2%	7.0%
75M	8.5%	8.0%	7.4%
62F	6.5%	6.5%	6.5%
67F	6.8%	6.8%	6.8%
75F	7.7%	7.5%	7.2%

[34] Authors estimates from data available from PC Bond Analytics, online at http://www.canadianbondindices.com/ (accessed 2010-4-8)

[35] Actually, modified durations should be matched, but the error introduced by the approximation is minor. Immunization is a bond portfolio management technique in which portfolio characteristics are selected in order to ensure that fixed cash payments at fixed times in the future can be met, regardless of market yields when coupon payments and maturities must be reinvested. The limiting form of immunization is cash-flow matching, in which strip bonds would be purchased to defease each individual cash flow.

[36] Sun Life Financial, *Annual Report 2008*, available on-line at http://www.sunlife.com/static/global/files/Annual%20reports/2008/pa_e_2008AR_annual_report.pdf

[37] JPMorgan, *Lifemetrics*, available on-line at http://www.jpmorgan.com/pages/jpmorgan/investbk/solutions/lifemetrics

[38] Lowell Aronoff and Moshe A. Milevsky, *Will your SWiP Beat Your ILY?*, 2008-12-15, available on-line at http://www.ifid.ca/pdf_newsletters/PFA_2008DEC_ILY.pdf (accessed 2010-3-26)

Annuities as Part of a Retirement Financial Plan

It should be apparent from the above discussion that annuities are a lousy investment. A seventy-five year old man can expect to achieve a yield of only 2.12% given average life expectancy. Even if he considers himself particularly healthy relative to his peers the expected returns are not particularly impressive, since the wickedness of the world is such that everybody else has already thought of that trick and the insurance companies have incorporated their adverse selection risk in their pricing.

They are a lousy investment, but they are great insurance! As shown in Chart 6, an investor buying an annuity at age 75 will, given current prices, have achieved a return of over 9% if he lives to be 95; an impossible feat to duplicate in the markets with the same level of investment risk. A retail investor has no chance to hedge his longevity risk using JP Morgan's LifeMetrics suite of products discussed above (which in any case deal with large statistical pools rather than individuals); any investor who requires more income than is generated by his portfolio and must therefore withdraw from capital is running the risk that his capital will disappear before he dies – a grisly fate, to be avoided if at all possible.

Additionally, the annuities discussed in this essay are all fixed rate annuities – there is no inflation protection afforded by these products. In fact, a prescribed annuity, with its preferential taxation regime, may not be indexed under current law.[39] To a certain extent, the effect of inflation is to implement, willy nilly, the suggestion that rational investors should choose to reduce consumption with age,[40] but the lack of inflation protection implies that an investor must retain some non-annuitized assets expected to perform well during inflationary periods.

Given these considerations – and even without considering the problem of loss of liquidity discussed earlier under the heading "Annuities" – it should be clear that a rational investor should address his risks through annuities, but not buy more than is necessary to assure that his portfolio, as a whole, meets his needs. But how much is that?

Kaplan derives an asset allocation model[41] for four asset classes: stocks, bonds, cash and annuities, with expected real returns for each asset class being as shown in Table 9, with an annuity payout rate of 7.5% for a 65-year-old male and inflation averaging 2.5%. This model incorporates inflation but ignores taxes. For comparison, note that current IFID data (discussed in the section 'Relative Pricing of Annuities') indicate a payout for a sixty-five year old man (with a ten-year guarantee) to be just under 6.6%. With a 5% withdrawal rate, the model results in annuitization of between 70% and 95% of initial assets.

Table 9: Kaplan's return assumptions (2005)

Asset Class	Real Return	Standard Deviation
Stocks	5.75%	17%
Bonds	3.25%	6.0%
Cash	1.5%	0.5%

The problem with this model – which is endemic in the efficient frontier literature – is that it treats fixed income returns as a (log-) normally distributed variable, which can only be the case when the time horizon is long compared to the term of the fixed income instrument selected. If our hypothetical 75-year-old retiree should invest in a thirty year government bonds, he knows exactly what the cash flows from that particular investment will be each and every year for the next thirty – which is an optimistically long time frame! Market volatility is only of interest to him to the extent that he needs to sell a portion of his holdings in this bond; if the coupon is sufficient for his needs, he may blithely ignore the day to day variance of the market, as shown in Table 3.

I will not pretend to have developed an efficient frontier model that addresses this concern. I will, however, suggest that PerpetualDiscount preferreds can be – under certain conditions – an effective substitute for annuities.

PerpetualDiscount Preferred Shares as Annuity Substitutes

Annuities are best considered when depletion of capital becomes a risk – that is, when expenditure exceeds investment income. Let us consider a 75-year old man allocating $100,000 principal value to an investment who requires after-tax income of $6,500 annually from this portion to maintain his lifestyle. He has narrowed down the choice to either PerpetualDiscount preferreds or an annuity, has other income and has addressed inflation risk elsewhere in his portfolio: this example is intended to address only the preferred vs. annuity decision. Data for this client are summarized in Table 10.

We define:

P_A = Principal Value invested in Annuity
P_P = Principal Value invested in Preferreds
I = After Tax Income

There is a total of $100,000 to be invested, so

$$P_A + P_P = 100{,}000 \quad (2)$$

and since required annual after-tax income is $6,500

$$I = 6{,}500 = 5.74\% * P_P + 10.46\% * P_A \quad (3)$$

And we then solve to find:

P_A = 16,100
P_P = 83,900

Table 10: Data for Notional Investor

Age	75 Years
Life Expectancy	10.26 Years
Portfolio Value	$100,000
Yield on Preferreds	6.00%
Marginal Taxation Rate on Dividends*	4.40%
After-Tax Yield on Preferreds	5.74%
Annuity Rate per $100,000	10,800
Taxable Portion of Annuity, per $100,000 invested	1,053
Tax Rate on Taxable Portion of Annuity*	32.50%
After-Tax Pay-out on Annuity (including ROC)	10.46%

**Marginal tax rates are as provided by Ernst & Young for 2009, for a BC resident with $75,000 taxable income; available on-line at http://www.ey.com/CA/en/Services/Tax/Tax-Calculators-2009-Personal-Tax (accessed 2010-4-8)*

[39] Fiscal Agents/Manulife Financial, *Annuity Taxation*, available on-line at http://www.fiscalagents.com/newsletter/4ca_rp_annuitytax.shtml (accessed 2010-4-8)

[40] Referred to in the section "Safe Withdrawal Rates"

[41] Paul D. Kaplan, *Asset Allocation with Annuities for Retirement Income Management*, April 2005, available on-line at http://www.ifid.ca/Conference_Material/2005_Kaplan_paper.pdf (accessed 2010-3-25)

The calculation is checked in Table 11:

Table 11: Solution Checking for Notional Investor

Amount invested in Preferreds	83,900
Pre-tax dividends from Preferreds (@ 6%)	5.034
Tax Paid on Preferred Income (@ 4.4%)	221
After-Tax income from Preferreds	4,813
Amount invested in Annuity	16,100
Pre-Tax Annuity payments (@ 10.8%)	1,739
Taxable Portion of Annuity Payments (@ 9.75%)	170
Tax Paid on Annuity Payments (@ 32.5%)	55
After-Tax Annuity Payments	1,684
Total Annual Cash After Tax	**6,497**

Note, however, that this calculation does not include an allowance for the higher risk of preferreds, which will be discussed in the next section; I suggest that income produced from preferreds should be 10% higher than the income required to account for this risk.

Equation (3) then changes to:

$I = 6,500 = 5.17\% * P_P + 10.46\% * P_A$ (4)

In which the 5.74% actual after tax yield of preferreds has been reduced by 10%, to 5.17%, to account for risk. When equation (3) is solved, we find that the desired allocation to preferreds is 74,900, with 25,100 being annuitized. Such a portfolio will produce $6,925 in after-tax income, in excess of the $6,500 requirement; it would be prudent to reinvest the excess annual receipts as capital in the portfolio.

Some may consider this example trivial, since the cash shortfall using only preferreds (making no allocation to annuities) is only $760 on a $100,000 portfolio; the capital depletion rate with a portfolio comprised entirely of preferreds is less than 1% per annum at current rates and the investor has a negligible chance of exhausting this capital even under extreme assumptions for longevity and adverse market moves.

However, consider the case where the cash requirement from this portfolio is 9,500. We then adjust equation (4) and solve:

$I = 9,500 = 5.17\% * P_P + 10.46\% * P_A$ (5)

We find that we can invest $18,147 in preferreds, leaving $81,853 for the annuity, while meeting the investment goal of an after-tax withdrawal rate of 9.5% of initial principal and maintaining a slight excess in annual income to cover adverse risks for the preferred portion. While the amount may seem small, this is a significant amount of principal – 18% of the portfolio, nearly two years' after-tax income – which can be converted at any time into consumption or left to heirs, as circumstances and preferences dictate. This flexibility has been achieved at a cost of about $900 p.a. (when the 'risk cushion' is included in income), which I suggest is reasonable given the situation assumed. In many calculations (such as those performed by insurance salesmen!), full annuitization may be considered preferable because the gain in flexibility is assigned a value of zero.

Risks Incurred When Substituting PerpetualDiscount Preferred Shares for Annuities

Credit Risk:

Annuities are a far more senior claim on the insurers than preferred shares, especially since – as far as the insurers are concerned – an annuity is a senior claim on the operating company's assets, while a preferred share is a junior claim on the parent. It is entirely possible that in times of trouble, a preferred shareholder could get nothing while an annuity holder could get paid in full … even in the absence of a government bail-out.

It should be noted that a holder of investment grade preferred shares is highly unlikely to learn suddenly that the company will default (although this is by no means impossible). It is much more likely that the decline will be gradual and marked by a steady series of Credit Rating downgrades.

Each downgrade will normally be accompanied (and preceded) by a decline in price; and it is more often than not the case that this decline in price will, for a long period of time, exceed what might be rationally expected from a rigorous examination of the credit quality (since many investors will choose to sell, exerting downward pressure on the market. Additionally, one may rely on market pundits to nod wisely at each other and discuss their "cockroach theory"). Following their downgrade from Pfd-2 to Pfd-3(high) in March, 2003, Bombardier preferred shares were cheap (relative to comparable instruments) for about a year, until the company had ceased to dominate the daily headlines.

The potential for downgrades and the desire to maintain overall portfolio credit quality by selling (not necessarily immediately) the downgraded instrument to purchase higher quality assets is the primary reason that a cushion of dividend income 10% in excess of requirements was recommended in the examples of annuity substitution by preferred shares. The loss of 10% of initial dividend income in an investment-grade portfolio would be a very severe and unexpected shock – particularly if, as recommended, this excess is reinvested rather than withdrawn upon receipt.

Call Risk:

Unfortunately, preferred shares are not true perpetual annuities; they are all subject to calls at par at some point. Hence, there is the risk that they will be called during a period of low yields. This will have the following ill effects:

- There will be capital gains tax to pay (since the PerpetualDiscount preferred shares were, by definition, purchased at a discount to par)
- There is no guarantee that suitable replacement investments will be available
- A call will normally take place when the issuer can refinance at lower rates, so replacement investments (including annuities) may yield less.

In order to minimize Call Risk, an investor should normally favour Straight Preferreds trading at a deep discount to par; but this bias should be informed by a comparison to comparable instruments trading with different coupons at different discounts. The effect of the issuers' call options on expected yield of PerpetualDiscounts was discussed in the January, 2010, edition of this newsletter.

Tax Risk:

The tax regime for dividends could change, eliminating at least some of the dividend advantage. Taxation of annuities could also change, but this is a less important risk due to political considerations; I suggest that any major changes would include a grandfathering of existing annuity contracts.

Inflation Risk:

This will be about the same for both annuities and Straight Preferreds, but you do have the option to buy an indexed annuity (which, however, will not qualify for the favourable "prescribed" treatment), whereas there are no indexed preferred shares at present. At some point, a deeply discounted FixedReset with a microscopic spread against five-year Canadas might be functionally equivalent, but we don't have any of those yet. Other floating rate perpetuals (Ratchet, FixedFloater, Floater) might be considered equivalent, but then you have basis risk (either prime or five-year Canadas vs. inflation) and extant non-FixedReset Floating Rate issues don't have sterling credit quality.

Sequence of Returns Risk:

In the examples above, the portfolio was constructed to avoid principal withdrawals from market instruments and hence make the sequence of returns irrelevant. However, investors who over-weight preferred shares and meet cash needs through partial sale will incur sequence of returns risk, which will exacerbate Principal Depletion Risk.

Principal Depletion Risk:

With an annuity, the insurance company takes the risk that you will last longer than expected, and covers it with their chances that other clients will make up for it. With a preferred share portfolio – or any investment portfolio – you're the one stuck with that risk. It is for this reason that the examples given have, essentially, made the required capital withdrawals via the purchase of an annuity.

Other Potential Annuity Substitutes

The correlation of annuity payouts with long corporate bonds suggested Preferred Shares as an annuity alternative, but it is equally possible to hold corporate bonds themselves. It will be recalled that Hymas Investment Management does not recommend that preferred shares comprise more than half of a fixed income portfolio; primarily due to the periodic paroxysms that shake the market, and the risks (such as Credit Risk, Call Risk and Tax Risk) that affect the preferred share market to a different degree than experienced by corporate bonds.

Those seeking to hold long corporate bonds specifically may be interested in the Bank of Montreal's BMO Long Corporate Bond Index ETF,[42] trading on the Toronto Exchange with symbol ZLC. This ETF is "designed to replicate, to the extent possible, the performance of the DEX Long Term Corporate Bond Index, net of expenses", and currently holds 44 issues.[43] It should be noted, however, that this ETF has been in existence for less than three months and has only about $4.5-million under management; its future existence is by no means assured.

Similarly, Mid-Term corporates could be held via BMO Mid Corporate Bond Index ETF,[44] trading with the symbol ZCM on the Toronto Exchange.

As always, specific investment products mentioned in this newsletter are neither endorsed nor deprecated, unless otherwise stated. They are merely mentioned, so that readers interested in the stated portfolio goals may be aware of the possibilities and determine whether the specific products meet their investment needs.

Naturally, it is also possible to own corporate bonds directly, but this can be an extremely frustrating experience. The selection of bonds available for retail investors is severely restricted compared to institutional offerings and, when one needs to sell, the client is at the mercy of a single bond desk. During the height of the Credit Crunch, many issues were "no bid" (sometimes even for institutional investors!) and even at the best of times a retail desk's bids are likely to be rather low.

However, a judicious selection of available bonds accumulated over a relatively long period of time, backed by core holdings of bond ETFs and mutual funds can allow the tailoring of a portfolio to suit individual needs.[45] Hymas Investment Management is pleased to serve as a consultant throughout the portfolio construction process.[46]

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[42] BMO Financial Group, *BMO Long Corporate Bond Index ETF*, available on-line at http://www.bmoetfs.com/ETFConsumer/controller/funddetails/glance?fundId=75747 (accessed 2010-3-28)

[43] BMO Financial Group, *BMO Long Corporate Bond Index ETF*, available on-line at http://www.bmoetfs.com/ETFConsumer/controller/funddetails/holdings?fundId=75747 (accessed 2010-3-28)

[44] Described on-line at http://www.bmoetfs.com/ETFConsumer/controller/funddetails/glance?fundId=75744 (accessed 2010-4-8)

[45] E.g., James Hymas, *The Bond Portfolio Jigsaw Puzzle*, Canadian Moneysaver November/December 2009, available on-line at http://www.himivest.com/media/moneysaver_0911.pdf

[46] See http://www.himivest.com/consulting.php

Hedging an Eventual Annuity Purchase

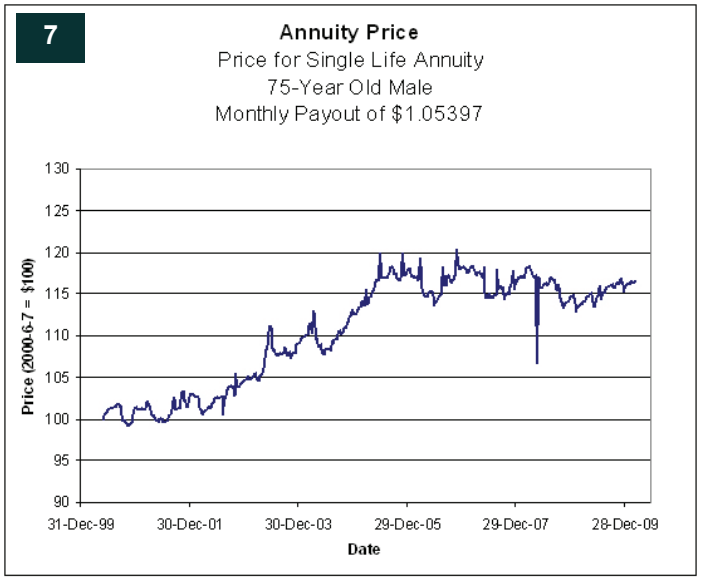
There is no need to wait until an annuity has actually been purchased before buying a preferred share or a corporate bond!

In the section "Absolute Pricing of Annuities" it was shown that, as a rule of thumb, the annual payout ratio on a prescribed single life annuity for a 75-year old man could be estimated as the Long Corporate yield plus 575bp and analysis of Milevsky's Implied Longevity Yield (discussed in "Relative Pricing of Annuities") suggests that other annuities may be priced using this figure in conjunction with the Long Corporate Yield.

Thus, it is entirely rational for an investor in his accumulation phase, who intends to purchase an annuity at some point in his retirement, to hedge this purchase using long corporate bonds and PerpetualDiscount preferred shares.

Many investors deprecate long bonds of any nature, dismayed by the price volatility reported in their brokerage statements; an attitude I have never been able to understand due to its inconsistency. I have previously remarked[47] that a five year GIC has exactly the same price volatility as a five-year Canada bond; the major difference is that market prices of Canada bonds are reported on brokerage statements, while GICs are almost always reported at historical cost (sometimes with accrued interest). I will now observe that the price volatility of an annuity is equally unreported.

Chart 7 shows the change in price of an annuity over time, recording the dollar price over time for a seventy-five year old male to purchase a prescribed single-life annuity for a constant monthly pay-out. The chart, prepared from IFID data,[48] shows a huge increase in price over time, as corporate yields have declined dramatically from their levels at the start of the index data in June 2000 … and what goes up may certainly come down.



Consider Dr. Milevsky's prototypical sixty-five year old male, determining whether to purchase an annuity immediately, or to wait ten years. The Implied Longevity Yield on 2010-3-3 was 5.91%; this investor might be tempted to purchase a ten year strip bond yielding in excess of 5.91% and rest comfortably in the erroneous belief that he has locked in excess return.

Unfortunately, the computation of ILY assumes that the future price of the annuity for the seventy-five year old does not change. A drop in yields (increase in annuity price) could very well occur and he might find that the annuity purchased with the strip-bonds proceeds on maturity pays less than his anticipated $904.35 monthly. The strip bond was a poor hedge, because its expected value in year ten (of par) was very poorly correlated with the expected price of the future annuity. Naturally, yields could have increased in the interim and made our investor a winner – but seeking to accomplish gains of this nature is speculating, not hedging.

A proper hedge will vary with expected price; the investor should have bought long corporate bonds. Provided the initial term to maturity is in excess of 20 years, they will still be long corporate bonds in ten years' time (a long bond is understood to have a remaining term in excess of ten years) and their yields will be relevant to the payout yield of the future annuity.

If yields increase, then there will be a capital loss on the position, certainly. But the expected payout on a standard annuity will have also increased, so that the monthly payout on the annuity paid with the proceeds of bond disposition will be the same as expected.

It should be well understood that hedges are never perfect. We have estimated that the payout of the standard "M75" annuity is equal to long corporate yields plus 575bp, and the hedge is only as good as the estimate. The chance that this relationship will change is referred to as "basis risk", which accompanies, to a greater or lesser extent, any investment hedging decision.

Naturally, our investor could also attempt to increase his yield during the accumulation or waiting period by buying PerpetualDiscount preferred shares, which are currently trading to have an interest-equivalent yield of more than three-hundred basis points in excess of long corporates. Each investor must decide whether this excess yield is sufficient for him to assume the risks of a preferred share investment (as discussed above) together with the increased basis risk of the hedge. The Seniority Spread (interest-equivalent yield on PerpetualDiscounts, less the yield on long corporates, as discussed in the October, 2009, edition of this newsletter), currently in excess of three-hundred basis points, could be less than this when the target date approaches (good!) but it could be more (bad!). It should be borne in mind, however, that the investment liquidity gained due to an investment in marketable instruments such as preferred shares has the effect of giving the investor more options: on the target date, or at any other date he chooses, he may repeat his calculations and decide whether the investment strategy needs adjustment. There is no necessity to buy an annuity on the date you circled ten years ago.

[47] James Hymas, *Preferred Shares and GICs*, Hymas Investment Management, August 2009, available on-line at http://www.himivest.com/media/PrefsAndGICs_090814A.pdf

[48] IFID, *Payout Annuity Index*, supra

Duration of an Annuity

It is all very well to blithely state, as we did above, that there is a relationship between long corporate yields and annuity prices, but in order to construct a proper fixed-income hedge, we must understand – as a very first step – the duration[49] of the instrument we are trying to match.

The data in Table 7, which used the insurance companies' expectations of their annuity payouts, imply that the duration of an annuity is about 6.7 years. However, not only could this analysis be faulty, it is also not directly relevant to the problem facing an investor who is accumulating a portfolio earmarked for the purchase of an annuity – or, at least, the potential purchase of an annuity; the great thing about liquidity is that you always have the opportunity to change your mind according to circumstances. This investor is not concerned about the insurance companies' problems, he is much more interested in what he will be charged for his prescribed, single life, no-guarantee annuity at the age of 75.

Chart 7 showed the price of this annuity as it changed over time; and a similar time series for long corporate bond yields was displayed in Chart 4. Given prices and contemporary yields, we can calculate the Effective Modified Duration of the instrument – at least to a first approximation.

A scatter plot of all available data is shown in Chart 8, and it is immediately apparent that something is wrong – if there is a good relationship between long term corporate yields and the annuity price the data should be much better distributed. However, it looks as if – as a preliminary hypothesis – there are two relationships in the data and that at some point in the course of data collection there has been a change from on to the other; something that is described formally as regime shifting.[50]

However, we have encountered this regime shift previously, in the discussion of absolute pricing of annuities. Chart 4 presented data in such a way that a regime shift in mid-2008 was clearly indicated; this was supported by the regression data shown in Table 4 and – very important if you wish to build a quantitative model with predictive ability, rather than merely mining the data – we are aware the Lehman bankruptcy and market chaos commencing in the third quarter of 2008 caused many breaks in market data.

Accordingly, Chart 9 is prepared, using only the data to 2008-6-30 and it supports the hypothesis: while the correlation between the plotted variables is by no means perfect, the bifurcated relationship has disappeared and a close relationship seems apparent.

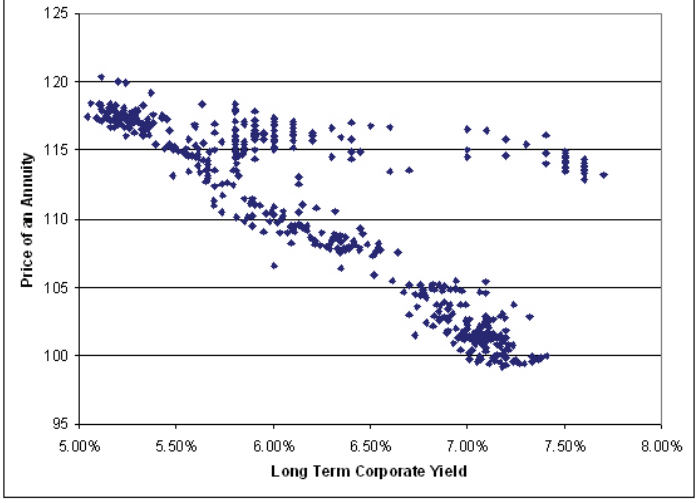
Regression data corresponding to the two charts is shown in Table 12. The data to 2008-6-30 indicate, with a good degree of confidence, that the price of an annuity is dependent to a very high degree (92% of the variance is explained) upon long corporate yields; and that the sensitivity to changes in this yield corresponds to a Modified Duration of 8.87 years. I suggest that this result be used for investors seeking to hedge the purchase of an annuity, rather than the lower figures derived in the section "Annuities as an Investment". It is, after all, the price of an annuity that we are seeking to hedge; not the value of an annuity, with all its embedded costs that are not considered in the derivation of the lower number. It would be interesting to play with the numbers, adding expenses and adverse selection adjustments until agreement was reached, but this must await another essay.

Using data from Table 7, we see that we can match this Modified Duration by constructing a portfolio with a weight of about 50% in Mid-Term Corporate bonds and the remainder in either Long-Term Corporate bonds or PerpetualDiscount preferreds. Such a portfolio will now yield between 5.15% and 6.65%, depending on the proportion of preferred shares. Note that the latter figure, reflecting a full allocation to PerpetualDiscounts, is interest-equivalent and applies only to taxable accounts.

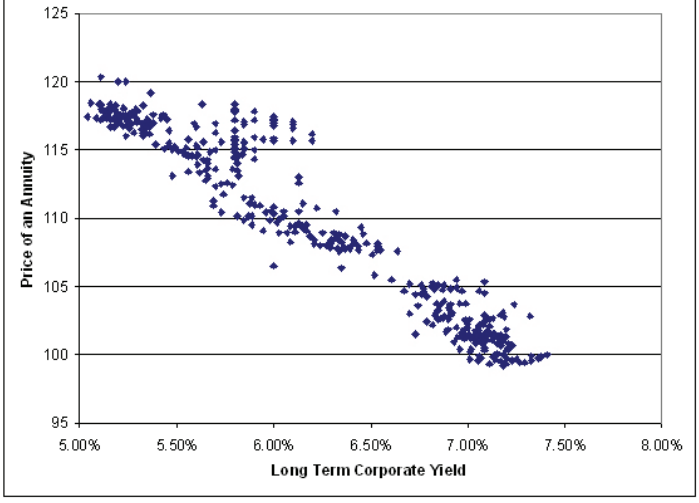
Table 12: Results of Regression
Annuity Price/100 = (slope) * Corporate Yield + (intercept)

Value	All Data to 2010-3-3	Data Prior to 2008-6-30 only
R-squared	0.54	0.92
Intercept (corporate yield = 0)	1.52	1.64
Slope	-6.43	-8.87

8 Annuity Calculation: All Data



9 Annuity Calculation: Data to 2008-6-30



[49] For more on duration, see James Hymas, *Modified Duration*, Canadian Moneysaver May 2008, available on-line at http://www.himivest.com/media/moneysaver_0705.pdf

[50] The informal description uses naughty words.

Conclusion

Investors seek to maintain a comfortable – or at least predictable – level of income throughout retirement, but planning is hamstrung by the fact that they do not know the period of consumption that is to be funded. This uncertainty as to the length of their retirement is referred to as Longevity Risk.

In this essay I have discussed the problems inherent in maintaining an adequate income throughout retirement, given the vagaries of investment markets. Any calculation of a "safe withdrawal rate" must include some estimate of the degree of principal depletion risk; many of the factors affecting this degree of risk (Longevity Risk and Market Risk) are entirely beyond an investor's control.

Annuities are popular because they explicitly eliminate principal depletion risk; this risk is transferred to the insurance company, which relies on statistical mortality rates to ensure that they can offer this service while covering their expenses and making a profit.

Thus annuities can be considered to be insurance; the investor is insuring himself against his longevity risk. However, while annuities are excellent insurance vehicles, they are poor investments. It is difficult for investors to 'play the odds' and relate the decision of whether to buy an annuity to his self-determined evaluation of longevity risk, because insurance companies expect such behaviour (from their point of view, this is adverse selection risk or, less formally, the "lemons problem") and incorporate these expectations into their pricing structure.

Annuities are insurance; insurance costs money; annuities should be purchased only to the extent necessary to insure the portfolio's Principal Depletion Risk.

It has been shown that the rates paid on annuities are closely related to the yields on long term corporate bonds, which in turn are related to the yields on PerpetualDiscount preferred shares; however, these latter two investment vehicles do not carry any insurance against Longevity Risk. Investors for whom longevity risk is a problem should seriously consider addressing this risk through the purchases of annuities, while remembering that these vehicles have a cost, both in terms of expected investment returns and liquidity. Thus, purchase of annuities should be restricted to the minimum amount necessary to ensure that capital is not depleted.

Investors who are accumulating funds for retirement, or who are retired and are contemplating the future purchase of annuities, should understand that the relationship between annuity pricing and long term corporate bond yields carries important implications for their asset allocation strategy if they seek to ensure a long term stream of income.