Convexity

James Hymas

In the May 2007 edition of *Canadian MoneySaver*, I noted that "the Modified Duration of a bond will increase as yields decline since the value of the principal repayment, relative to the value of the interim coupon payments, will increase. The degree to which Modified Duration is dependent upon Yield is known as convexity, and by convention this relationship is expressed as a positive value."

Positive convexity is the normal case for bonds and is a useful property for them to have. For a bond with no embedded options, positive convexity means that:

- As the price of the bond decreases, the yield will increase – the relationship is expressed as Modified Duration ("ModDur").

- As the yield increases the ModDur will decrease – the relationship is expressed as convexity.

Putting these two statements together leads to the insight that there is at least one way in which fixed-income mathematics works in a way that is favorable to investors: As the price goes down, the sensitivity of price changes to further yield increases also goes down. As the price increases due to yield declines, it accelerates. This is what we like to see!

In some cases, however, a fixed-income instrument can have a negative convexity; this is a bad attribute but, as with so much in the investing world, it is something that we might be willing to put up with as long as we're paid for it.

Two issues will be examined in detail: SBN.PR.A, which is very "bond-like", and PWF.PR.D,

which is not very bond-like at all, at least not at current market prices – a point which will be discussed later.

SBN.PR.A has only a single mechanism whereby the company forces the return of capital to its lenders, a maturity on December 1, 2014. Readers may verify this through examining the prospectus, available either through SEDAR at http://www.sedar.com or through the sponsor, Mulvihill, at http://www.mulvihill.com.

PWF.PR.D, however, is a perpetual issue that is just entering its redemption period. It may be called at $26.00 commencing 2007-10-31; the price declines by $0.20 annually until 2012-10-31 at which point it may be called at $25.00 forever afterwards. There are no retraction privileges (for a discussion of the differences between perpetual

TABLE 1: CALCULATION OF MACAULAY DURATION AND CONVEXITY FOR A BOND PAYING A 5% ANNUAL COUPON AND PRICED AT PAR TO YIELD 5%.

Date Cash Received	Amount (A)	Time (T)	Discounting Factor (DF = (1 / 1.05)^T)	Present Value (PV = A · DF)	PV · T	PV · T · (1+T)
May 1, 2008	$1.25	1	0.9524	$1.1905	1.1905	2.3810
May 1, 2009	$1.25	2	0.9524^2 = 0.9071	$1.1338	2.2676	6.8028
May 1, 2010	$1.25	3	0.9524^3 = 0.8639	$1.0799	3.2397	13.3188
May 1, 2011	$1.25	4	0.9524^4 = 0.8228	$1.0285	4.1140	20.5700
May 1, 2012 (Interest)	$1.25	5	0.9524^5 = 0.7836	$0.9795	4.8975	29.385
May 1, 2012 (Maturity)	$25.00	5	0.9524^5 = 0.7836	$19.5901	97.9505	587.7030
			Totals	$25.0023 (Rounding Error!)	113.6598	660.1606
				Macaulay Duration	4.5460	
					Years (= 113.6598 / 25.0023)	
					Convexity	23.95 (= 660.1606 / (25.0023 * 1.05^2))

These calculations have been performed as of May 1, 2007. It has, therefore, been assumed that there is no accrued interest on the bond. When accrued interest is calculated as part of the total settlement price, the total of the present values will include the accrued interest; the price effect of yield changes predicted by Modified Duration will then refer to the change in the total value of the instrument, not just its quoted price.

and retractible issues, see the June 2006 edition of *Canadian MoneySaver*). The issuer has published a summary of the issue specifications on its website at http://www.powerfinancial.com. For more details, check SEDAR.

When calculated for a normal bond, that is to say, one without embedded options, convexity is always positive in accordance with the formula:

$$C = \sum (PV \cdot T \cdot (1+T)) / (P \cdot (1 + i)^2)$$

Where: C is convexity,

PV is the present value of each cash flow,

T is the time until the cash flow is received,

P is the total price of the bond (equal to the sum of PV), and

i is the rate of interest.

Those readers with a mathematical bent will instantly recognize the similarity of this equation to the equation for ModDur given in the May 2007 edition of *Canadian MoneySaver*. This only makes sense since convexity measures the rate of change in ModDur as the price changes. A convexity calculation for a normal bond is shown in Table 1.

SBN.PR.A is mathematically equivalent to a normal bond since it has no embedded options. Any cash-flow analysis will be as simple as we can expect a preferred share analysis to be. And when we look at Figure 1, the graph of ModDur vs. Price for this issue, we see that ModDur does in fact increase smoothly with price. So, all is well.

When we look at PWF.PR.D on the other hand, with ModDur vs. Price plotted in Figure 2, we see a different pattern entirely. While ModDur increases with price over parts of the plot, the overall move is quite violently nega-

**FIGURE 1 - INSTRUMENT: SBN.PR.A (SECURITY A48880)**
X- Axis: Bid, Y-Axis: Modified Duration - YTW

FIGURE 2 - INSTRUMENT: PWF.PR.D (SECURITY A44002)
X- Axis: Bid, Y-Axis: Modified Duration - YTW
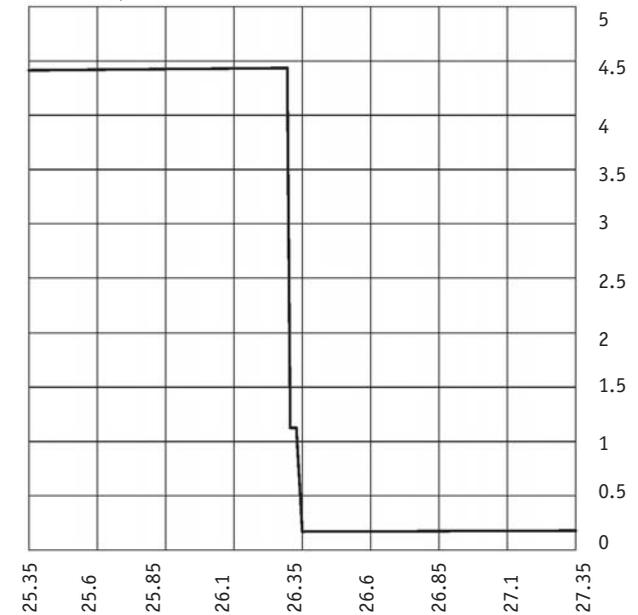
tive. This implies that sensitivity to interest rates will (to a large extent) increase as interest rates increase.

The overall implications of this behaviour will be familiar to those conversant with "Perpetual Hockey Sticks", discussed in the March/April edition of *Canadian MoneySaver*: When the issue is trading at a high price above the call price, it is less sensitive to interest rate moves than when it is trading below the call price. This implies that we can group perpetual issues into three subgroups: those trading at a significant discount to their redemption price, those above, and those that are within range of this price. Note that I have been deliberately vague about "range" because it will depend on the individual investor.

So, which type of perpetual is best suited for an investor? The answer, as always in the case with investments, is "it depends". But I will attempt to provide some broad rules to flesh out the article on portfolio construction from the July/August edition of *Canadian MoneySaver*.

Our first task is to quantify negative convexity. This is a very difficult thing to do. Although traditional bond mathematics, as shown in Table 1, provides a very efficient method of quantifying convexity, it is simply not possible to adapt this procedure to evaluate the convexity of fixed-income instruments with embedded options. In order to provide some quantitative figure that may be fed into HIMIPref™ (my firm's preferred share modeling software), my firm has developed two major methodologies of estimation:

• **Pseudo-convexity** - With this technique, the modified duration is calculated at three different prices: a price 1% greater than the actual price, the actual price, and a price
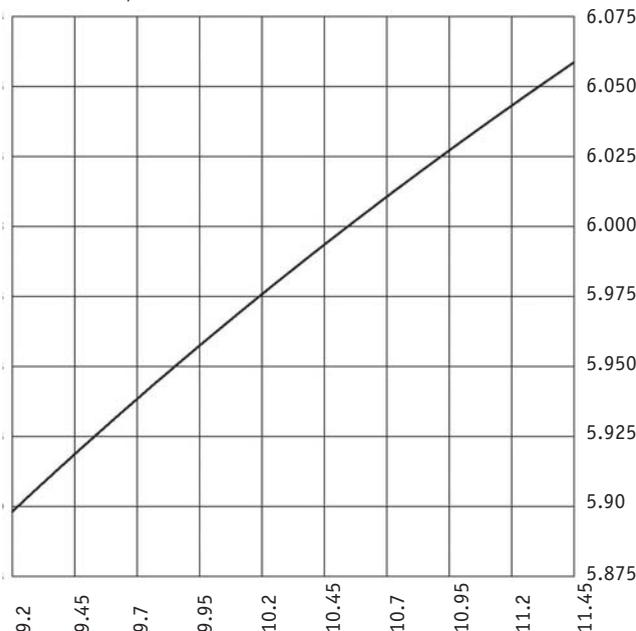
1% lower. These three ModDurs are denoted MD_{i+1}, MD_i, and MD_{i-1}, respectively. We then calculate the percentage change in ModDur over the range as $\Delta MD = (MD_{i+1} - MD_{i-1})/ MD_i$. We also calculate the percentage change in price over the range, $\Delta P = (P_{i+1} - P_{i-1})/ P_i$. The pseudo-convexity, C_{pseudo} relates the change in ModDur to the change in price: $C_{pseudo} = \Delta MD / \Delta P$. This technique of taking values at different prices and simply plugging these values into further formulae is not original. Used for duration, for example, when we want to know how the yield changes with price, it has been referred to as "Effective Duration" or "Key Rate Duration".

• **Option Doubt** - I will confess that I conceived, programmed, and was half-way through the testing of this parameter before I realized it was a measure of convexity! HIMIPref™ calculates probabilities that options will be exercised, which results in each instrument being viewed as a portfolio of scenarios. There might be a 10% chance of option exercise at $26.00 in one year and a 90% chance of exercise at $25.00 in five years, for example. The "Option Doubt" parameter is simply the weighted standard deviation of the calculated terms to option exercise. Thus, when only one option is considered possible (an extremely high-premium perpetual being considered certain to be called at the first possible date, for example), Option Doubt is zero. When there are many options with a wide range of exercise dates (a currently callable perpetual trading at par could last one day or forever), Option Doubt is large. It takes some thinking about, but the results of the calculation give results very similar to pseudo-convexity.

Neither of these methods is perfect. A number of approximations are made for computational expediency and computational expediency has an awful habit of making programmers look silly when large numbers are divided by small ones. But these methods have proven to be of great practical use, and that's always the main thing!

Analysis by my firm indicates that the difference between a "bond-like" perpetual trading at about 15% above or below its redemption price is worth about 15bp more in yield than one trading at par, which has negative convexity, indicating that it is not very bond-like. In other words, if new issues arrive priced at par with a dividend yield of 5.25%, they may be considered equivalent, all else being equal, to either a deeply discounted or a high premium perpetual from the same issuer yielding 5.10%.

With a deeply discounted perpetual, we have the chance of making a good-sized capital gain before the issuer calls away our holding. With a high premium perpetual, we have relative certainty about the date on which it will be called away. Either situation is preferable to having an issue priced at par, which suffers the worst of both worlds – and it should

take about 15bp of yield to reconcile a cautious investor to this disadvantage.

James Hymas, CFA, Hymas Investment Management, Toronto, ON, (416) 604-4204, jiHymas@himivest.com, http://www.himivest.com/ , http://www.prefshares.com/.