Modified Duration

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I have made frequent references to "Modified Duration" in this column over the past year; so much so that I should explain the term and its importance to fixed income in general and preferred shares in particular.

We all know instinctively that a thirty-day bill has less exposure to interest rate risk than a thirty-year bond—should interest rates spike, the former will lose much less market value. But how much less? Modified Duration is a handy—and surprisingly easy to calculate—measure of this interest rate sensitivity. It has its faults; it is dependent upon various assumptions that may not always hold true but it is a superb first approximation to defining the interest rate risk of a particular fixed-income investment.

The relevant equation is:

$$\Delta P = - D_{Mod} \cdot \Delta I \qquad \text{(Equation 1)}$$

Where : ΔP is the percentage change in price
ΔI is the change in percentage yield
D_{Mod} is the Modified Duration

Not bad! Once we know the Modified Duration of an instrument, we will know how much the price will change when the yield changes by a specified amount—at least, for as long as the assumptions used when computing Modified Duration are valid. And I might as well let you know right up front: they're never valid. Virtually all fixed-income mathematics is a little fraudulent that way, but as long as you know the limitations of the analysis, it's still worthwhile.

In this column, we'll calculate the Modified Duration of a bond that pays a coupon of 5% once per year, commencing exactly 1 year from now and maturing in exactly five years.

In order to calculate the Modified Duration, D_{Mod}, we first need to calculate the Macaulay Duration, D_{Mac}, since:

$$D_{Mod} = D_{Mac} / (1 + (y/f)) \qquad \text{(Equation 2)}$$

Where : D_{Mod} is the Modified Duration
D_{Mac} is the Macaulay Duration
y is the yield-to-maturity of the instrument
f is the number of payments per year.

$$D_{Mac} = (\textstyle\sum PV_i \cdot T_i) / (\textstyle\sum PV_i) \qquad \text{(Equation 3)}$$

Where : PV_i is the Present Value of the i'th cash flow
T_i the time the i'th cash flow is received
i is just a counter that identifies each one of our cash flows

And finally, Present Value is the amount of money that, if received right now and invested at the yield of the instrument (y, in Equation 2) would be equal to the amount required at the required time (T, in Equation 3).

Table 1 (see next page) shows a calculation of the Macaulay Duration of a fictitious bond—with very conveniently chosen numbers!—and derives a value of 4.55 years. By substitution of values into Equation 2 we find:

$$\begin{aligned} D_{Mod} &= 4.55 / (1 + (0.05/1)) \\ &= 4.55 / 1.05 \\ &= 4.33 \end{aligned}$$

Since the Modified Duration is 4.33, we claim that according to theory, a change of 0.1% in absolute interest rates will change the price of the instrument by 0.433%. This claim is checked in Table 2 (see next page), where we find that the price of the bond that equates to a 5.1% Yield-to-Maturity is 25.1093, while a yield of 4.9% equates to a price of 24.8921. These values are, respectively, 0.44% above and 0.43% below the price that yields 5.0%. Success! Within limits, anyway. As a change in yield gets larger, Equation 1 gets less accurate. Like everything else in fixed-income mathematics, it works every time, except for when it doesn't.

Changing bond characteristics will change Modified Duration, and as long as we don't strain the math too much, these effects are easy to understand once the basic theory is out of the way:

● **Term To Maturity** : An increase in term-to-maturity will increase Modified Duration of a bond valued at par (since the PV·T terms of Equation 3 will be multiplied by a greater number, while the divisor, which is the total price of the bond, will remain constant). However, this effect will decrease as the term gets larger, until at terms-to-maturity of
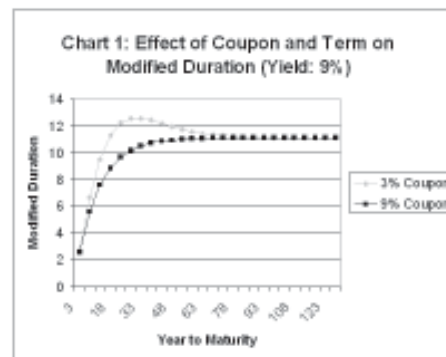
TABLE 1 - CALCULATION OF MACAULAY DURATION FOR A BOND PAYING A 5% ANNUAL COUPON AND PRICED AT PAR TO YIELD 5%.

Date Cash Received	Amount (A)	Time (T)	Discounting Factor (DF = $(1 / 1.05)^T$)	Present Value (PV = A·DF)	PV·T
May 1, 2008	$1.25	1	0.9524	$1.1905	1.1905
May 1, 2009	$1.25	2	0.9524^2 = 0.9071	$1.1338	2.2676
May 1, 2010	$1.25	3	0.9524^3 = 0.8639	$1.0799	3.2397
May 1, 2011	$1.25	4	0.9524^4 = 0.8228	$1.0285	4.1140
May 1, 2012 (Interest)	$1.25	5	0.9524^5 = 0.7836	$0.9795	4.8975
May 1, 2012 (Maturity)	$25.00	5	0.9524^5 = 0.7836	$19.5901	97.9505
Totals				**$25.0023** (Rounding Error!)	**113.6598** 4.5460 Years (= Macaulay Duration 113.6598 / 25.0023)

These calculations have been performed as of May 1, 2007. It has therefore been assumed that there is no accrued interest on the bond. When accrued interest is calculated as part of the total settlement price, the total of the present values will include the accrued interest. The price effect of yield changes predicted by Modified Duration will then refer to the change in the total value of the instrument, not just its quoted price.

Chart 1: Effect of Coupon and Term on Modified Duration (Yield: 9%)

duration principal.

• **Coupon Payments**: A plot of Modified Duration against time for a single bond with constant yield will show a saw-tooth pattern, with Modified Duration declining steadily until a coupon payment results in an upwards jump. This is because the smallest contributor to the PV·T term, the next coupon, will disappear when the coupon is paid.

• **Coupon Frequency**: A careful examination of Table 1 should convince any doubters that an increase in coupon frequency will decrease Modified Duration. The bond analyzed in the table pays annually. If it paid semi-annually, then the sum of the PV·T terms would decline, since half of the coupon would be paid six months earlier than otherwise. Quarterly payments—and, for some preferreds, monthly payments—will increase this effect.

• **Yield**: The Modified Duration of a bond will increase as yields decline since the value of the principal repayment, relative to the value of the interim coupon payments, will increase. The degree to which Modified Duration is dependent upon Yield is known as convexity, and by convention this relationship is expressed as a positive value. This is a crucial concept when discussing the structure of a bond portfolio and when discussing preferreds…so crucial, in fact, that I won't discuss convexity much in this article but leave it for another time.

more than 30 years (at current interest rates) Modified Duration will only barely react to changes in term. This is because the Present Value of the principal payment is relatively small at these terms.

• **Coupon Rate**: If several bonds have the same term-to-maturity and the same yield-to-maturity, higher coupon issues will have a lower Modified Duration than lower coupon issues (they will also have a higher price, of course). This is because the PV·T terms will assign a greater relative weight to the shorter-duration coupons than to the longer-
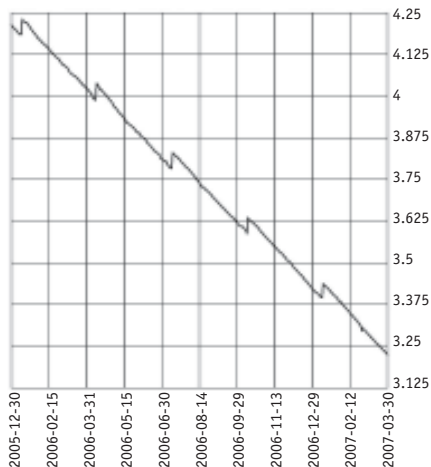
Even when the mathematics works perfectly, there can be some counter-intuitive results when calculating Modified Duration. Chart 1 shows the Modified Duration of two bonds, identical except for their coupon rates, as calculated when we extend term. Intuition works fine if we just look at the left side of the graph: Modified Duration increases with increasing term, more so for the lower coupon issue. But the lower coupon issue shows declining Modified Duration afterwards! This is due to the fact that at

TABLE 2 - CALCULATION OF THE PRICE OF THE BOND IN TABLE 1 WHEN YIELD IS VARIED

Date Cash Received	Amount (A)	Time (T)	Yield of 4.90%		Yield of 5.10%	
			Discounting Factor (DF = $(1 / 1.049)^T$)	Present Value (PV = A·DF)	Discounting Factor (DF = $(1 / 1.051)^T$)	Present Value (PV = A·DF)
May 1, 2008	$1.25	1	0.9533	1.1916	0.9515	1.1894
May 1, 2009	$1.25	2	0.9088	1.1360	0.9053	1.1316
May 1, 2010	$1.25	3	0.8663	1.0829	0.8614	1.0768
May 1, 2011	$1.25	4	0.8258	1.0322	0.8196	1.0245
May 1, 2012 (Interest)	$1.25	5	0.7873	0.9841	0.7798	0.9748
May 1, 2012 (Maturity)	$25.00	5	0.7873	19.6825	0.7798	19.4950
Totals				**25.1093**		**24.8921**

**CHART 2 - PIC.PR.A - PROPERTIES FROM
2005-12-30 TO 2007-03-30**
X-Axis - Date
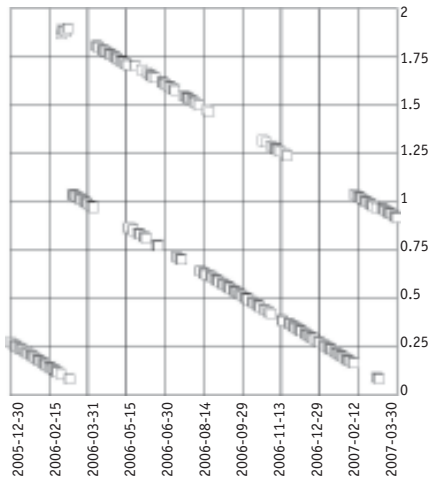Y-Axis - Modified Duration - YTW

the relative short terms, a huge proportion of the bond's value is represented by repayment of principal—and the Modified Duration will increase rapidly with increasing time until the principal is repaid; in other words, the bond is behaving very much like a zero-coupon bond. At longer terms, however, the discounting applied to the principal repayment negates this effect by reducing the Present Value of the principal to a negligible amount, the effects of the coupons take over, and the bond's Modified Duration behaves much like a perpetual annuity.

Let's get back to preferred shares for a moment and consider the plot of Modified Duration vs. time for PIC.PR.A, shown in Chart 2. This split share issue was briefly discussed in the November 2006 edition of *Canadian MoneySaver* ("Split Shares"), and a quick glance at the characteristics of the issue at www.prefinfo.com shows that it has a single maturity date with no embedded options. Very bond-like, in fact! The plot shows three characteristics that will be present in such plots of any bond (barring changes in credit and huge swings in price):

- Overall, downward sloping over time, since the Modified Duration declines as the term to maturity declines (although see Chart 1 for one exception!).
- Sudden small rises in Modified Duration, when the coupons are paid and the cash flow is ignored rather than used to lower the average term-to-maturity of the cash flows.
- A certain amount of unevenness in the plot—not particularly visible in this instance, I'm afraid, but you can't have everything—as day-to-day price changes imply yield changes, which impact the calculations.

PIC.PR.A is an example of an issue where the terms of the issue do not violate the assumptions of the math. However, as stated previously, classical fixed-income mathematics (whence Macaulay Duration and Modified Duration are derived) has some problems. One problem is the assumption of a flat yield curve—did you notice? In Tables 1 and 2, the yield used to derive the discounting factor is the same for each coupon, and we know that's not right. In the real world, each term has its own particular yield—which is why the so-called "Zero-Coupon" or "Spot-Yield" curve is so important in precise work.

Fortunately, the preferred share market (remember the preferred share market? That's what this column is supposed to be about.) is so inefficient that worrying about the precision of classical fixed-income mathematics with respect to classical bonds is not necessary. A far greater problem that must be dealt with when using Modified Duration with preferred shares is the effect of embedded options.

As an example of this, we'll consider ABK.PR.C. This issue is callable at $60.80 on March 10 of every year, with a final maturity of March 10, 2008. On the 2006 redemption, approximately 6.5% of the issue was redeemed, while in 2007 about 13.4% of the shares then outstanding were redeemed. Further information is available through the website of the sponsor, www.scotiamanagedcompanies.com.

**CHART 4 - ABK.PR.C - PROPERTIES FROM
2005-12-30 TO 2007-03-30**
X-Axis - Date
Y-Axis - Modification Duration - YTW

**CHART 3 - ABK.PR.C - PROPERTIES FROM
2005-12-30 TO 2007-03-30**
X-Axis - Date
Y-Axis - Yield-to-Worst (at Bid)

As shown in Chart 3, the presence of these embedded options has caused the Yield-To-Worst of this issue to vary significantly over the past year—to the point where it has from time-to-time gone negative! Of even greater interest is Chart 4, which shows the Modified Duration of the redemption scenario which results in the lowest yield. It is clear from this chart that there is more than one scenario which has been applicable over the past year and the calculated Modified Duration of the Yield-To-Worst scenario must be examined with a jaundiced eye. Only an examination of the other possible scenarios and the Modified Duration that applies to these scenarios will allow us to make an informed decision regarding the interest-rate sensitivity of any particular issue.

So, there you have it—an example of fixed-income mathematics when it works, and an example of when it doesn't. When dealing with preferred shares with embedded options, investors must not just examine the most likely redemption scenario, but also assess the effects that a switch to another scenario might have. In a future issue, I will discuss some of the mathematics developed by my firm that attempts to deal with these problems in a systemic manner.

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