Implied Volatility for FixedResets
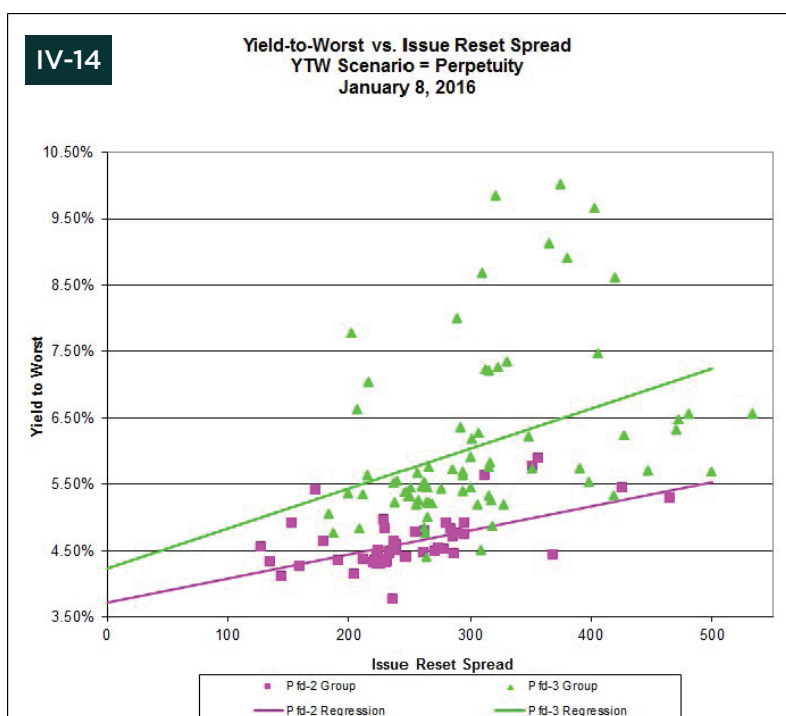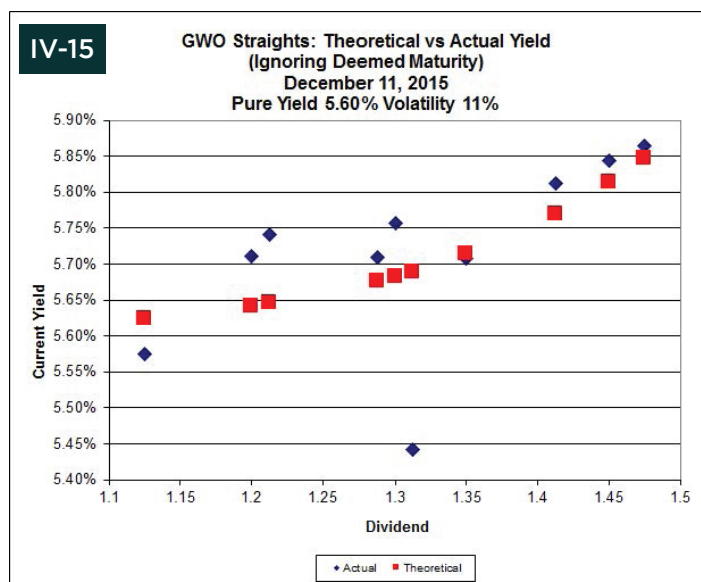
This essay was put together from material published in the September, 2013, and January, 2016, editions of PrefLetter.

Those who are familiar with the theory of Implied Volatility of Straight Preferreds[1] will be struck by the resemblance of Chart IV-14 (which plots Yield-to-Worst FixedResets against their Issue Reset Spread) to the implied volatility graphs for for Straight Perpetuals (see Chart IV-15) – an increase in the underlying dividend is well correlated to an increase in the expected yield to perpetuity for instruments in the class, even after market forces have re-priced the issue.

This is no accident, as the mechanisms are precisely the same: Increasing the Issue Reset Spread (with all else held constant) of an instrument for which a call is unlikely means an increasing price which

  i) Reduces the potential for capital gain

  ii) Increases the uncertainty regarding the term of the issue and therefore

  iii) Should result in an increased yield to compensate for these factors.

To this end, an algorithm for determining the Implied Volatility of a series of FixedResets has been developed, as explained in the earlier part of this essay. The calculator that gives effect to this algorithm has been made public via http://www.prefblog.com/xls/ImpliedVolatility.xls. It will be noted that the "Expected Future Current Yield" is the Current Yield that will result if each issue resets at its spread over the presumed GOC-5 level of 0.75% (or any other user input) and the price remains constant.

IV-15

GWO Straights: Theoretical vs Actual Yield
(Ignoring Deemed Maturity)
December 11, 2015
Pure Yield 5.60% Volatility 11%

IV-14

Yield-to-Worst vs. Issue Reset Spread
YTW Scenario = Perpetuity
January 8, 2016

In this essay I explain the calculation of Implied Volatility for FixedResets, in accordance with a spreadsheet I have developed which is publicly available at http://www.prefblog.com/xls/ImpliedVolatility.xls. The spreadsheet is compatible with MS-Excel 2003, and must be downloaded to your hard drive in order to work.

The Black-Scholes Option Model

In order to price the embedded call, we may use the Black-Scholes option pricing model, a complex mathematical formula based on sometimes dubious assumptions. A good overview of the technique is available at http://bradley.bradley.edu/~arr/bsm/model.html.

Briefly, the Black-Scholes model assumes that the best estimate of a financial instrument's future price is its current price, but that the actual future price will vary around this estimate in a well-defined way – a bell curve. The price of an option will be determined by the chance that it will be valuable at the time of its expiry: if you have an option with a 10% chance of being worth a dollar and a 90% chance of being worthless; the option's price should be ten cents (ignoring adjustments for the time value of money).

The width of the bell curve of possible future prices is dependent upon two factors: the time to expiry of the option (naturally, the longer to expiry, the wider the distribution) and the volatility – a measure of how much change in the price may be expected in a standard period of time (usually a year).

Volatilities do not have to be expressed in terms of price; it is entirely admissible to perform the calculation in terms of yields, which then provides results in terms of yields.

Additionally, it may be shown[2] that: *The interpretation of N(d1) is a bit more complicated. The expected value, computed using risk-adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is N(d1) multiplied by the current stock price and the riskless compounding factor. Thus, N(d1) is the factor by which the present value of contingent receipt of the stock exceeds the current stock price.*

The present value of contingent receipt of the stock is not equal to but larger than the current stock price multiplied by N(d2), the risk-adjusted probability of exercise. The reason for this is that the event of exercise is not independent of the future stock price. If exercise were completely random and unrelated to the stock price, then indeed the present value of contingent receipt of the stock would be the current stock price multiplied by N(d2). Actually the present value is larger than this, since exercise is dependent on the future stock price and indeed happens when the stock price is high.

Note, however, that the calculation of N(d2) and N(d1) in the spreadsheet are based on the Issue Reset Spreads, not on price; this formulation is much more likely to be normally distributed in practice than any calculation that involves historical prices or the GOC-5 yield.[3]

Data and Calculations Performed in the Spreadsheet

On the spreadsheet note that green cells contain semi-permanent data, updated infrequently; yellow cells contain fitting data, updated by the user whenever an observation is made; and purple cells contain calculations (and, in the case of column B only, external data).

Ticker: Column A, semi-permanent data: This ticker symbol takes the format specified by Yahoo! so that bid prices for all instruments may be easily loaded.

New Bid: Column B, external data: This is the most recent bid price on the Toronto Stock Exchange, as downloaded on the tab labeled "Yahoo".

Spread: Column C, semi-permanent data: This is the Issue Reset Spread, expressed in basis points.

Annual Dividend: Column D, calculated data: This is the dividend rate, in dollars, to which the issue will reset on the next exchange date, given the Spread specified in Column C and the GOC-5 rate specified by the user.

Expected Current Yield: Column E, calculated data. This is the Current Yield of the instrument, given the Annual Dividend calculated in Column D and the Price provided in Column B. It will be noted that this approximation introduces an error into the calculation – ideally, the actual current dividend would be provided as semi-permanent data and the difference between this value and the Annual Dividend calculated in Column D would be multiplied by the time until the actual Reset Date (which would also have to be provided). This difference between the dividends received according to the calculation and the dividends legitimately expected to be received would be used to adjust the bid price – possibly with an adjustment for the Present Value of the difference, rather than the undiscounted total. After consideration, I assumed that the error involved in the approximation used would be less than normal market noise, but I have not checked this assumption.

Pure Price: Column G, calculated data. This is the value of an uncallable Perpetual Annuity paying the annual dividend, discounted at a rate equal to the sum of the user inputs GOC-5 and Market Spread.

---

[2]   Lars Tyge Nielsen, *Understanding N(d1) and N(d2): Risk-Adjusted Probabilities in the Black-Scholes Model,* available on-line
      at http://www.ltnielsen.com/wp-content/uploads/Understanding.pdf (accessed 2013-9-7)

[3]   I was challenged at one point on this issue. See http://prefblog.com/?p=27695#comment-193179
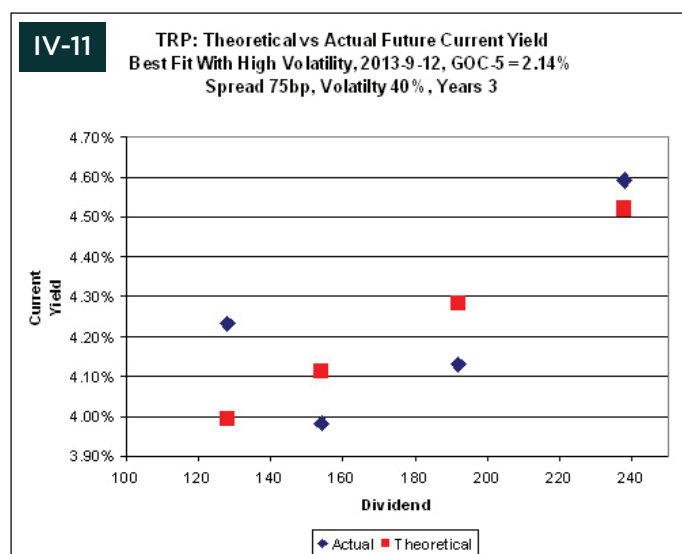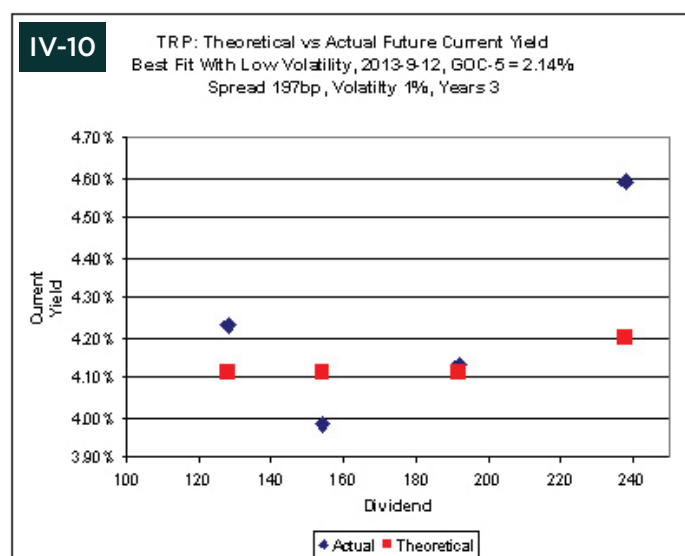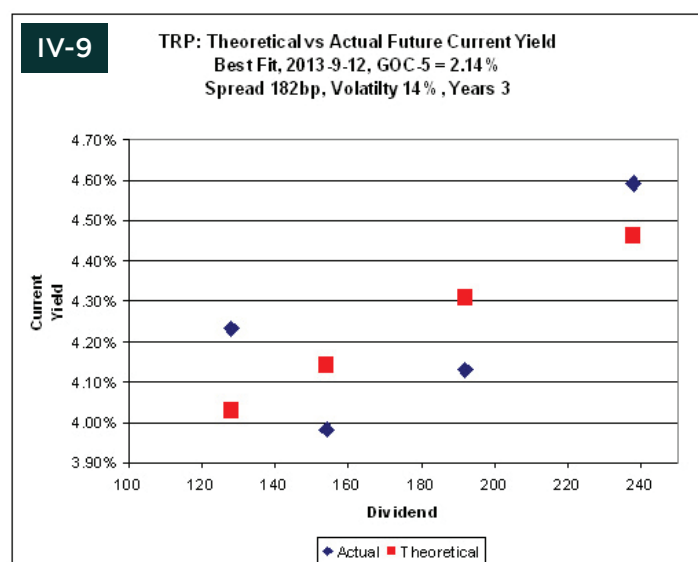
Option Adjustment: Column H, calculated data. This is simply the call premium calculated in Column Q. It is copied here for user convenience in reading the overall results of the calculation in columns G through L.

Theoretical Price: Column I, calculated data: This is the sum of the Pure Price (column G) and the Option Adjustment (Column H). This is the price at which the instrument should be trading given the market data provided by the user and the assumptions discussed in this section.

Squared Error: Column J, calculated data: This is the square of difference between the actual price (Column B) and the Theoretical Price (Column I), which is the error shown in Column L. The user is expected to minimize this error by varying the inputs marked in yellow.

Theoretical Current Yield: Column K, calculated data. This is the Expected Annual Dividend (column D) divided by the Theoretical Price (column I). It has no major importance, but is very useful for visualizing the relationships in the automatically generated chart.

Error: Column L, calculated data. The raw difference between the Theoretical Price (Column I) and the Actual Price (Column B). It is useful to be able to add the squared-errors of column J separately according to the sign of the raw error; this enables the user to center the theoretical prices by varying spread and then adjusting the slope of the relationship by varying volatility. An example of the effects of varying volatility shown in Charts IV-9, IV-10 and IV-11.

IV-9 — TRP: Theoretical vs Actual Future Current Yield, Best Fit, 2013-9-12, GOC-5 = 2.14%, Spread 182bp, Volatilty 14%, Years 3

IV-10 — TRP: Theoretical vs Actual Future Current Yield, Best Fit With Low Volatility, 2013-9-12, GOC-5 = 2.14%, Spread 197bp, Volatilty 1%, Years 3

IV-11 — TRP: Theoretical vs Actual Future Current Yield, Best Fit With High Volatility, 2013-9-12, GOC-5 = 2.14%, Spread 75bp, Volatilty 40%, Years 3

Term 1: Column N, calculated data: This is the first term of the Black-Scholes equation, which, as quoted from Nielsen, above, is: *The expected value, computed using risk-adjusted probabilities, of receiving the stock at expiration of the option, contingent upon the option finishing in the money, is N(d1) multiplied by the current stock price and the riskless compounding factor.* Specifically to this spreadsheet, this is the Pure Price (column G) multiplied by N(d1) (Column AC)).

e^-rt: Column O, calculated data: This is the discounting factor required for Term 2 (Column P) and is the equal to the base of natural logarithms ("e"), to the exponent of the negative of the risk free rate ("r", calculated in Column Z) multiplied by the term to option exercise ("t", from Column X)

Term 2: Column P, calculated data: This is the second term of the Black-Scholes equation, which, as quoted from Nielsen, above, is: *the present value of contingent receipt of the stock … when the stock price is high.* It is equal to e-rt (the discounting factor from column O) multiplied by N(d2), the cumulative normal distribution function calculated in column AD.

Call Premium: Column Q, calculated data. This is the value of the call option, equal to the difference between Term 1 (calculated in column N) and Term 2 (calculated in column P)

T Years: Column X, calculated data copied from user input. This is the term in years until option exercise and is simply copied from the user input data. It will be noted that in this calculation, T is the same for all instruments, which introduces an error into the calculation – since, of course, Exchange Dates in a set of FixedResets from the same issuer will normally be unique for each issue. I believe that this approximation will have little effect on the calculation compared to the normal vagaries of the market. As the market becomes more efficient, this approximation may have to be reviewed!

Sigma: Column Y, calculated data copied from user input. This is the volatility of the Market Reset Spread as input by the user.

RiskFree: Column Z, calculated data. This is the risk-free rate that is of great importance in the Black-Scholes formulation and here again I have made a design decision that some may consider to be less than optimal. The Black-Scholes model sets the Call Premium equal to the cost of delta-hedging exposure to securities; the risk-free rate is the amount paid to finance a long position, or received from the investment of proceeds of a short position.[4] In most expositions of the Black-Scholes theory, it is assumed that the cost of carry is the risk-free rate, since the underlying stock pays no dividends. This simplifying assumption is assuredly not the case with preferred shares, so the risk free rate is adjusted by the dividend yield on the preferred. In this spreadsheet I have assumed that

- The risk-free rate is the five-year GOC rate input by the user, and
- The Expected Current Yield from Column E is the benefit (cost) from being long (short) the stock; note that this ignores tax effects.
- The cost of carry is the difference between the two.

d1: Calculated value, Column AA: Calculated in standard Black-Scholes fashion from the Issue Reset Spread (column C), Market Spread (User input), risk-free rate (Column Z), Sigma (Column Y), and Term (Column Z). Here again is a design decision that some may see fit to criticize. Black-Scholes is normally calculated with stock prices, but in this case the stock price includes the option, which at the very least will make the calculations fearsomely calculated. It might be possible to use the Pure Price calculated in Column G, but in the end I decided to use the Issue Reset Spread (Column C) and the Market Spread (User Input) as the determinants of call probability. Note that this means that

- It is assumed that the Market Spread is log-normally distributed, and therefore
- Market Spread must be positive.

d2: Calculated value, Column AB: Calculated in normal Black-Scholes fashion from d1 (Column AA), Sigma (Column Y) and Term (Column X).

N(d1): Calculated value, Column AC: The Microsoft-Excel NORMSDIST function applied to d1 (Column AA)

N(d2): Calculate value, Column AD: The Microsoft-Excel NORMSDIST function applied to d2 (Column AB)

---

[4]   Joel R. Barber, *Delta Hedging with Black-Scholes Model*, available on-line at http://www2.fiu.edu/~barberj/s-chpt15.pdf (accessed 2013-9-12)   4

The Effect of GOC-5 On Theoretical Prices

All analyses in this section were performed with the following variables held constant:

Market Spread: 190bp
Volatility: 14%
Years: 3

Effect of GOC-5 on Pure Price

The Pure Price is the price of a perpetual annuity paying the indicated dividend, given the market yield.

Since:

$$\text{Market Yield} = \text{GOC-5} + \text{Market Spread} \qquad (1)$$

And

$$\text{Annual Dividend} = (\text{GOC-5} + \text{Issue Spread}) * 25 \qquad (2)$$

And

$$\text{Pure Price} = \text{Annual Dividend} / \text{Market Yield} \qquad (3)$$

Then substitute (2) into (3):

$$\text{Pure Price} = (\text{GOC-5} + \text{Issue Spread}) * 25 / \text{Market Yield} \qquad (4)$$

And substitute (1) into (4)

$$\text{Pure Price} = (\text{GOC-5} + \text{Issue Spread}) * 25 / (\text{GOC-5} + \text{Market Spread}) \qquad (5)$$
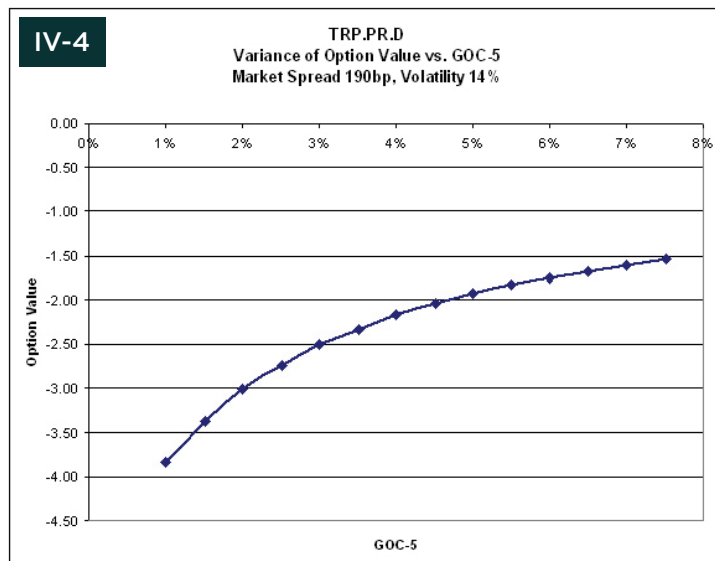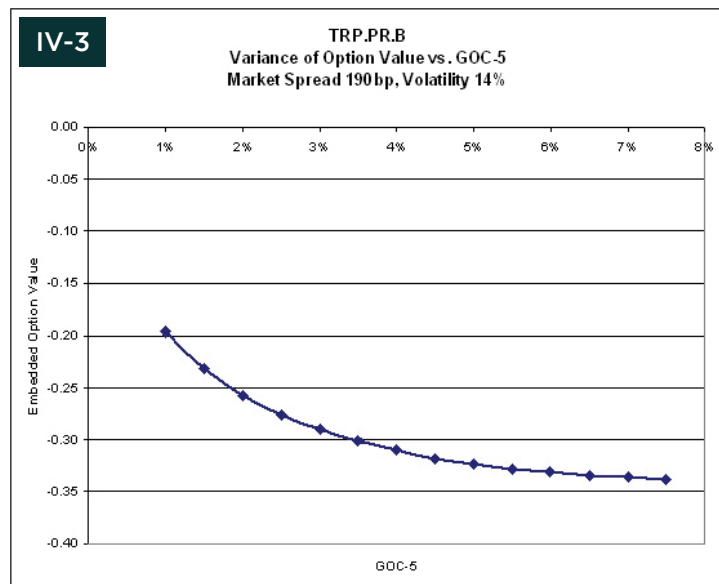
Clearly, therefore, as GOC-5 increases (theoretically until Issue Spread and Market Spread become so relatively small that they are effectively zero) the Pure Price will approach 25.00; this is illustrated for TRP.PR.B and TRP.PR.D in Charts IV-1 and IV-2.
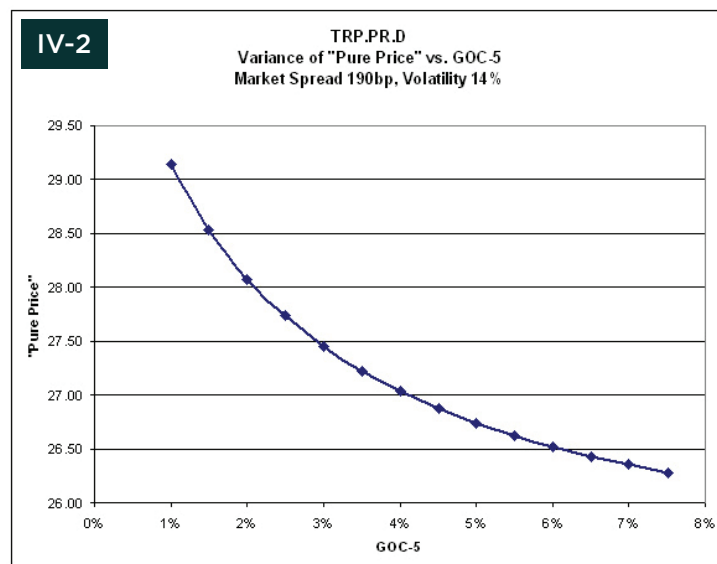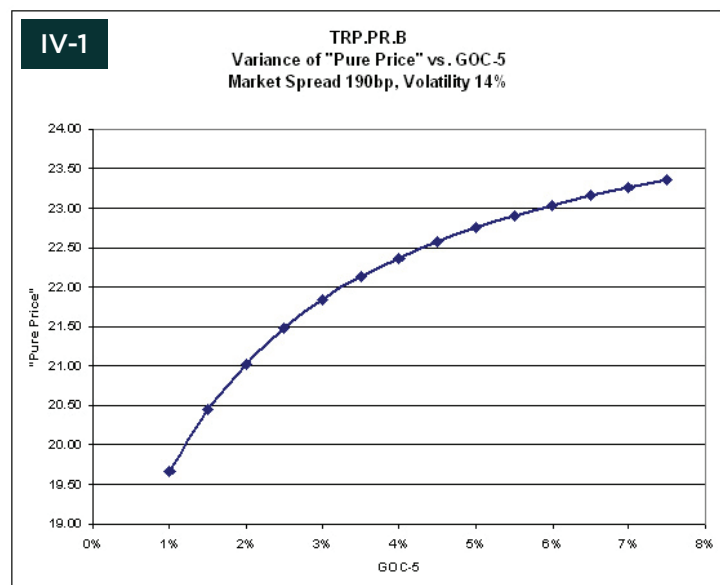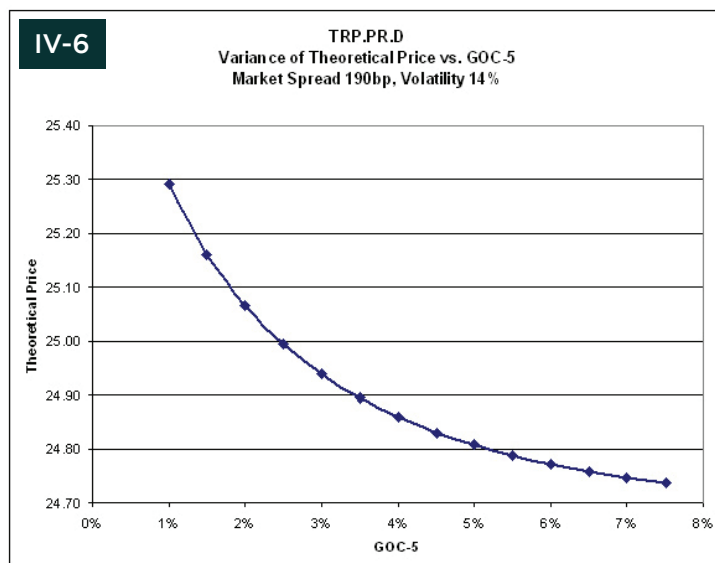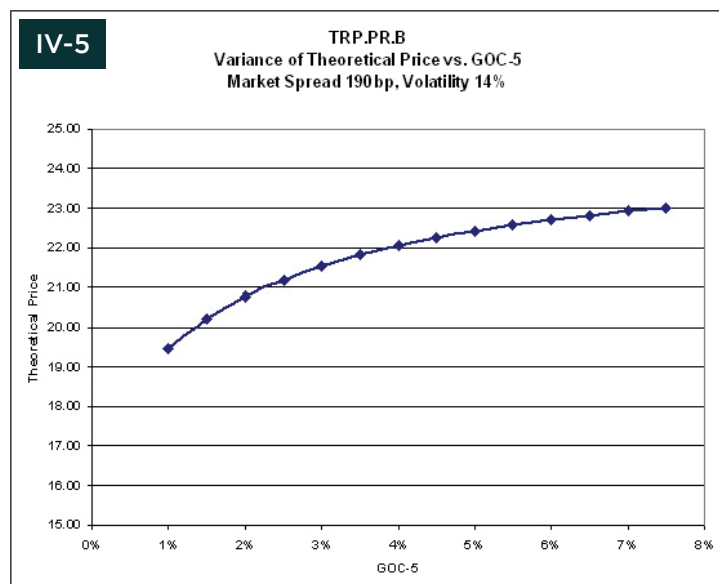
Effect of GOC-5 on Embedded Option Value

Once the Pure Price has been evaluated, we need to include the value of the embedded option in order to determine the Theoretical Price.

While the details are not especially intuitive, the general trend is clear: since the Pure Price will approach 25.00 as GOC-5 increases, the option value will decline if the Pure Price is above par, and increase if below, as shown in Charts IV-3 and IV-4.

IV-3 — TRP.PR.B — Variance of Option Value vs. GOC-5 — Market Spread 190bp, Volatility 14%

IV-4 — TRP.PR.D — Variance of Option Value vs. GOC-5 — Market Spread 190bp, Volatility 14%

Effect of GOC-5 on Theoretical Price

The theoretical price is the sum of the Pure Price and the embedded option value, which move in opposite directions as GOC-5 increases. The net effect of these trends is shown in Charts IV-5 and IV-6.

IV-5 — TRP.PR.B — Variance of Theoretical Price vs. GOC-5 — Market Spread 190bp, Volatility 14%

IV-6 — TRP.PR.D — Variance of Theoretical Price vs. GOC-5 — Market Spread 190bp, Volatility 14%

6

Effect of GOC-5 on "Expected Current Yield"

The "Expected Current Yield" is the dividend expected following reset at the indicated GOC-5 rate, divided by the current theoretical price.

As may be seen from Chart IV-7, these figures rise smoothly with increases in GOC-5; the slope is very slightly greater than 1; that is, an increase of 100bp in the absolute GOC-5 rate may be expected to cause an increase in Expected Current Yield of slightly less than 102bp for TRP.PR.B and slightly more than 102bp for TRP.PR.B.

Effect of GOC-5 on "Spot Current Yield"

An objection to the validity of Chart IV-7 is that changes in the GOC-5 rate are not, in fact, realized immediately, but take effect on the next Exchange Date. If the dividend rate on both instruments is held constant while varying the theoretical price in accordance with changes in the GOC-5 rate, we obtain Chart IV-8.

It will be noted that Chart IV-8 embodies an internal contradiction, as the calculation of both Pure Price and Option Value assume that changes in the GOC-5 rate are immediately reflected, whereas the "Spot Current Yield" calculated here assumes that they are not.

In fact, the Theoretical Price should be adjusted to reflect the difference in dividends received between the calculation date and the Reset Date, but I have opted not to do this in the interest of keeping the spreadsheets simple.
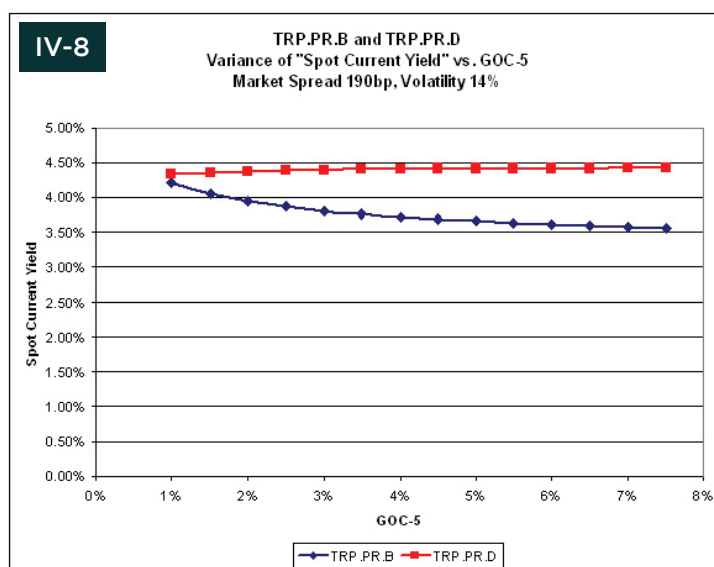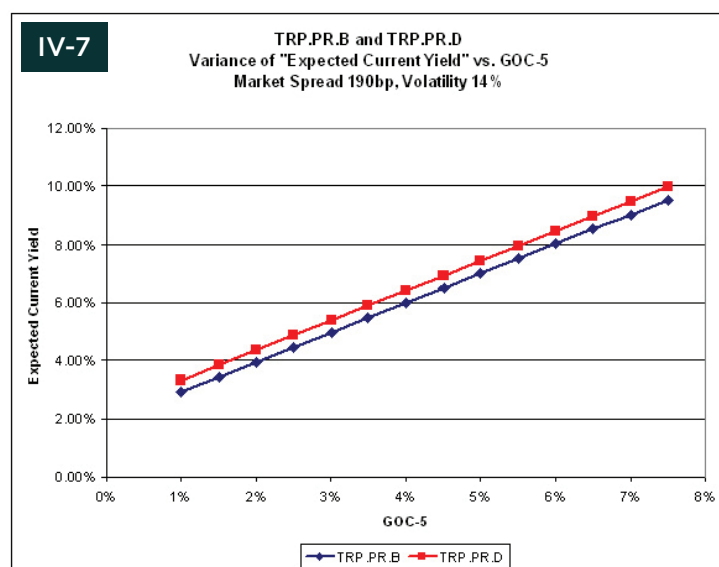
The Four TRP Issues in 2013

TRP.PR.D commenced trading on 2013-3-4; using the four issues on dates since that time provides the results shown in Table IV-1:

Table IV-1: Results of TRP Implied Volatility Calculations – Best Fit				
Date	GOC-5	Market Spread	Volatility	Sum Squared Error
2013-3-28	1.23%	61	27%	0.80
2013-4-30	1.12%	65	28%	0.91
2013-5-31	1.31%	53	32%	1.63
2013-6-28	1.74%	73	32%	1.67
2013-7-31	1.74%	84	32%	1.14
2013-8-30	1.91%	93	32%	0.79
2013-9-12	2.14%	99	32%	2.75

However, it must be noted that not only is the market a rather messy place in terms of pricing, but that local minima exist in the plotting – that is, if one considers a three-dimensional graph, with the x-axis being the Market Spread, the y-axis being the Volatility and the z-axis – the height – being the sum of square errors, there will be more than one point in which any small changes in the x, y, or any combination of the two, will result in an increased height.

I was unable to plot a contour map that showed this to my satisfaction, but the error was calculated for each combination of Market Spread and Volatility in the ranges 50-250bp and 1-40%, respectively. The calculations were performed using the bid prices for the four TRP FixedResets on 2013-9-13, with a GOC-5 rate of 2.12% and a term of three years.

The minimum value for each constant setting with varying values of the other setting was found and results are plotted in Charts IV-12 and IV-13. It is clear that the calculations are susceptible to local minima.

Table IV-2: Results of TRP Implied Volatility Calculations – Informed Fit				
Date	GOC-5	Market Spread	Volatility	Sum Squared Error
2013-3-28	1.23%	79	20%	1.23
2013-4-30	1.12%	86	20%	1.20
2013-5-31	1.31%	89	18%	2.18
2013-6-28	1.74%	127	16%	3.09
2013-7-31	1.74%	159	14%	1.98
2013-8-30	1.91%	175	13%	1.59
2013-9-12	2.14%	182	13%	3.35

I believe that this is particularly true in the current market, which is making something of a transition between pricing based on Current Yield with the expectation of a call on the first reset date and pricing based on the Issue Reset Spread and the expectation that issues will be left outstanding for perpetuity; but this, of course, is mere speculation.

In Table 1, the "best fit" of TRP data on various dates since the issue of TRP.PR.D, the volatility is unreasonably high – particularly considering that it is not the volatility of interest rates in general that is being quantified, merely the volatility of the Market Spread (this will include the term premium, the credit spread and the Seniority Spread, to borrow terminology usually used for PerpetualDiscounts).

More reasonable figures are shown in the results for local minima calculated in Table 2, but it will be observed that the Sum Squared Errors in Table 2 are greater than those of Table 1.

IV-12 — Minimum Error with Varying Volatility

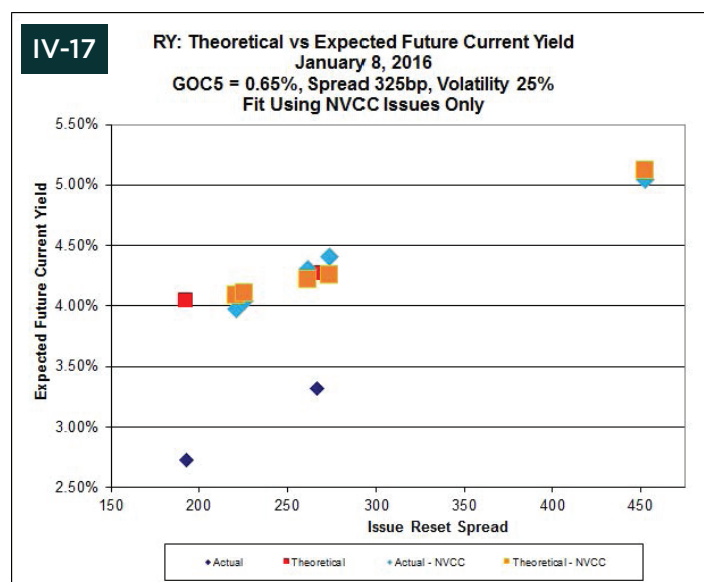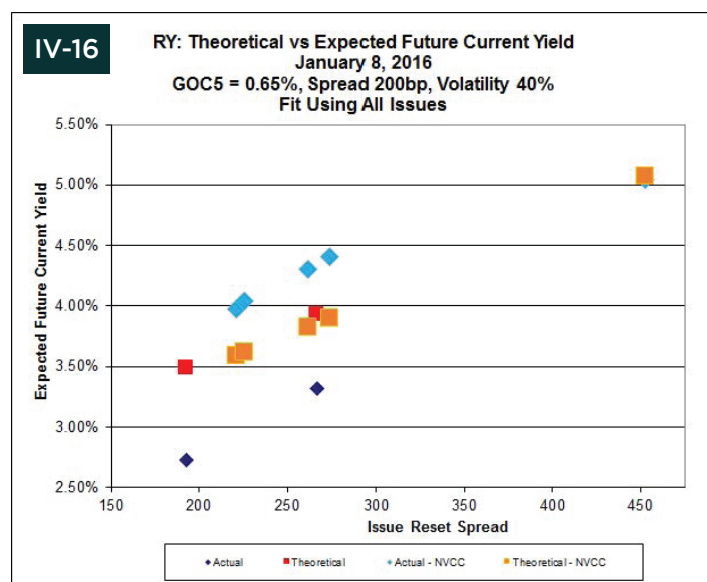IV-13 — Minimum Error with Varying Market Spread
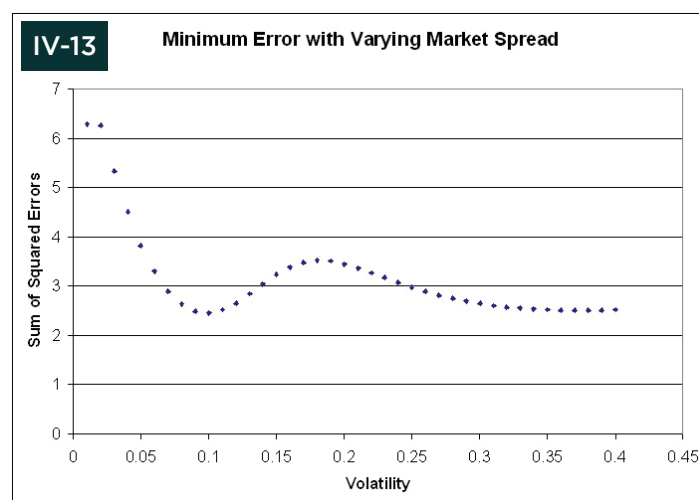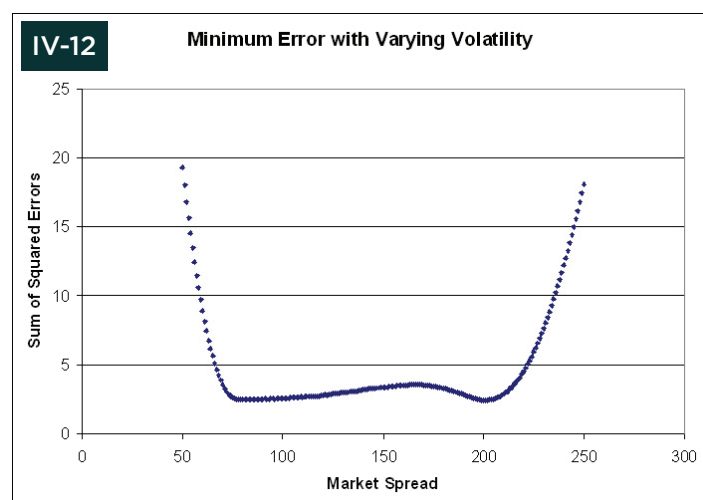
Implied Volatility of Market Spreads

Implied Volatility calculations for the entire set of Royal Bank (RY) FixedResets do not produce a reasonable result; even setting volatility to the maximum reasonable number of 40% does not succeed in matching the slope of the actual curve, as shown in Chart IV-16; but while the fit is much better when the fitting is restricted to the four NVCC-compliant issues (Chart IV-17), Implied Volatility remains at an unreasonably high level. Thus, while it is clear that the NVCC-compliant issues, RY.PR.Z, RY.PR.H, RY.PR.J, RY.PR.M and RY.PR.Q, have become differentiated from the NVCC-non-compliant issues, RY.PR.I and RY.PR.L, the NVCC-compliant issues suggest an unreasonably high value of Implied Volatility, which I interpret as an unwarranted assumption by the market that the future prices of these issues have a directional component.

However, there is another explanation for unreasonably high levels of Implied Volatility, albeit one that is probably only applicable to series with a significant number of low-spread issues. It is possible that there is a degree of preferential buying of low-spread issues due to speculators attempting to take advantage of these issues' high sensitivity to changes in the GOC-5 yield (see the section Effect of GOC-5 on Theoretical Price, above). This would cause the lower-spread issues in such series to increase in price relative to their higher-spread siblings, hence, lower the Expected Future Current Yield of these issues preferentially, hence increase the slope of the fitted curve, hence result in a higher calculated value of Implied Volatility. We will have to see how this all turns out, but I confess to being taken aback recently by a comment that said,[5] in part, *The implication for me of this is that currently longer dated resets with lower reset spreads … are very much a "coiled spring"*.

The Implied Volatility calculated for MFC issues (Chart IV-18) is 38% as of January 2016. This level is still too low if MFC issues are assumed to be subject (eventually!) to the NVCC rules but far too high for a series assumed not to be subject to these rules.

It will also be noted that the calculation for the TD series of NVCC-compliant FixedResets (Chart IV-19) also shows an unreasonably high level of Implied Volatility.

IV-16 — RY: Theoretical vs Expected Future Current Yield, January 8, 2016, GOC5 = 0.65%, Spread 200bp, Volatility 40%, Fit Using All Issues

IV-17 — RY: Theoretical vs Expected Future Current Yield, January 8, 2016, GOC5 = 0.65%, Spread 325bp, Volatility 25%, Fit Using NVCC Issues Only
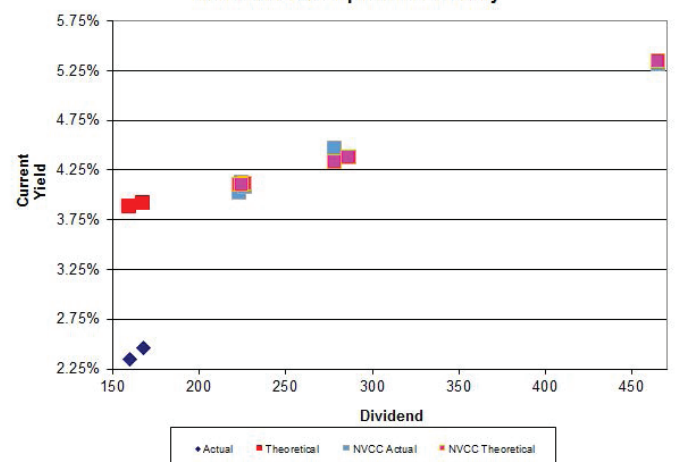

IV-18 MFC: Theoretical vs Expected Future Current Yield
GOC5 = 0.65%, Spread 266bp, Volatility 38%
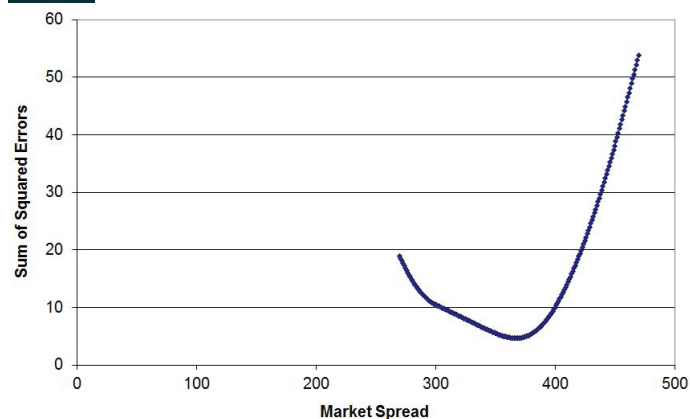January 8, 2016
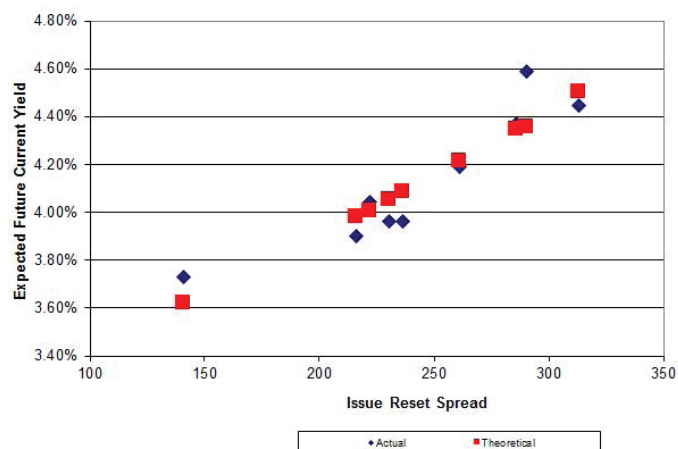
IV-19 TD: Theoretical vs Actual Future Current Yield
January 8, 2016
GOC5 0.65%, Spread 301bp, Volatility 32%
Fit for NVCC-compliant Issues Only

To my great relief, the local minimum that was apparent in early 2014 for the TRP issues (see Charts IV-12 and IV-13) has disappeared, as displayed in Charts IV-20 and IV-21 – it is clear that the fit, shown in Chart IV-22, is a true global minimum, although the fit isn't very good. It will also be apparent that the Market Spread has increased dramatically since the compilation of Table IV-2. This does not make a lot of sense, but was also seen during the Credit Crunch with respect to Floating Rate preferreds.[6]
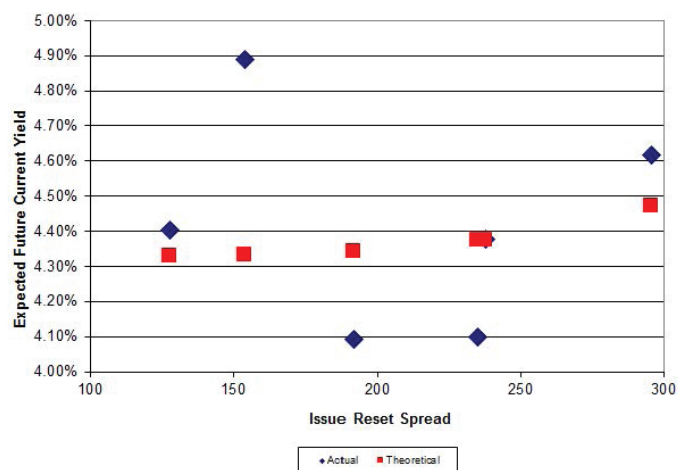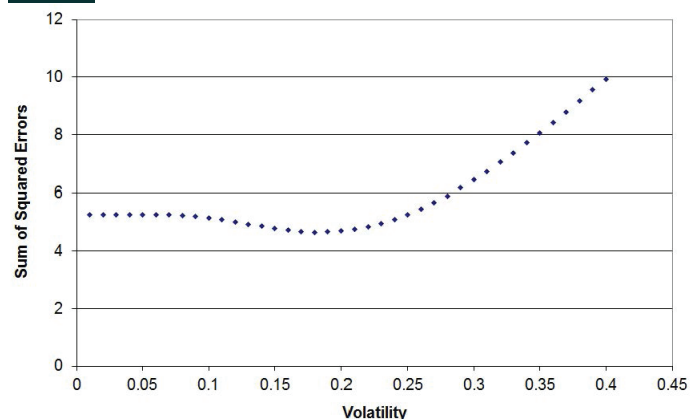

IV-20 Minimum Error with Varying Volatility

IV-22 TRP: Theoretical vs Expected Future Current Yield
January 8, 2016
Volatility 18%, Spread 368bp, GOC5 0.65%

IV-21 Minimum Error with Varying Market Spread

The calculated Implied Volatility of 18% is very high. The value of N(d2), the Risk Adjusted Exercise Probability (see the section *The Implications of High Volatility: Black-Scholes Option Pricing and N(d2)*, below) is about 2.8% for both TRP.PR.D (resetting at +238bp, bid at 17.31) and TRP.PR.E (resetting at +235bp, bid at 18.30) and about 11% for TRP.PR.G (resetting at +296, bid at 19.55).
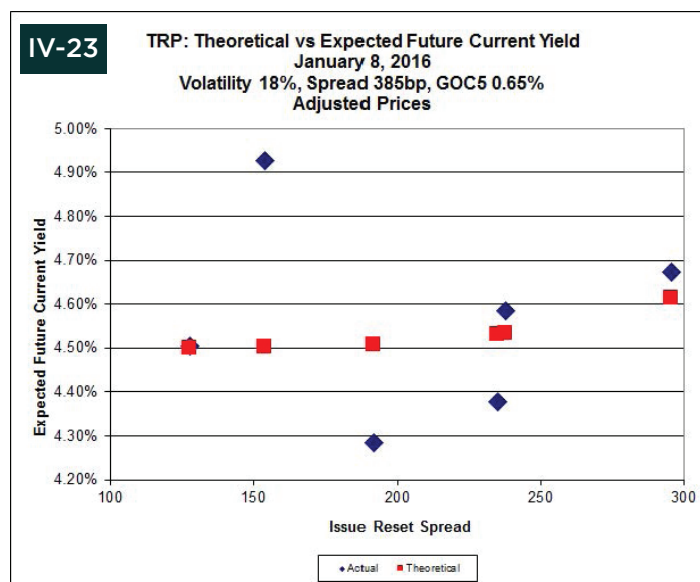
Some of the pricing differences may be ascribed to the fact that these calculations are performed using the expected long-term dividend rate, given a constant five-year Canada yield of 0.65%, when in fact the dividends will be paid at some other rate until the next reset. For instance, TRP.PR.E currently pays $1.0625 (4.25% of par), which is expected to reset to (0.65% + 238bp) * 25 = 3.13% * 25 = $0.7825 on its reset date, 2019-10-30. The difference is $0.28 p.a. and there are fifteen dividend payments left before reset, so the total excess payment is $1.05. We could therefore subtract $1.05 from the market bid price of TRP.PR.E for analytical purposes, treating the instrument as a package of a callable perpetual annuity of $0.7825 p.a. and a short term cash receipt of 1.05 (undiscounted). This calculation is summarized for each of the TRP issues in Table IV-3.
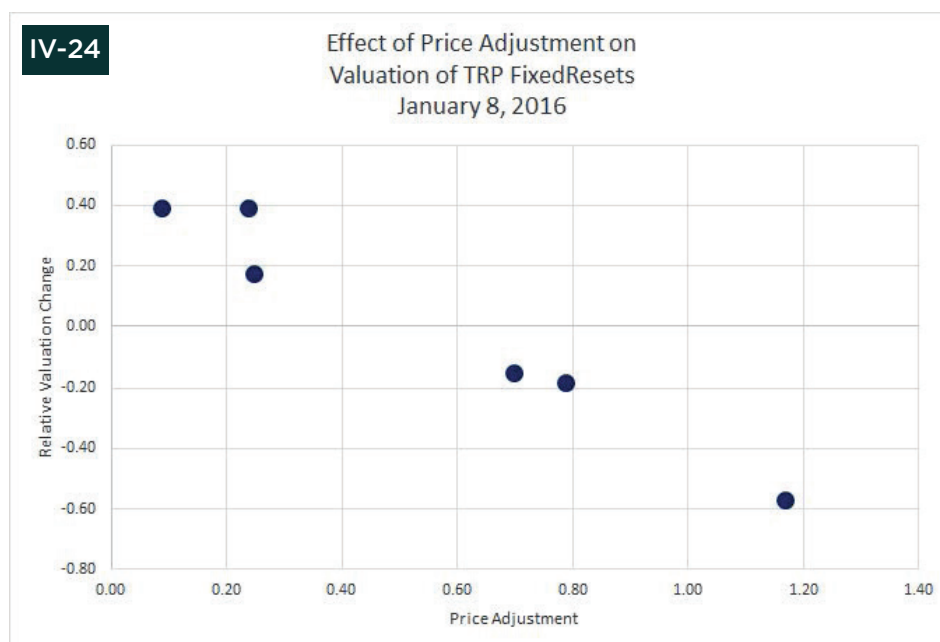
Table IV-3: Price Reductions of TRP FixedResets To Reflect Excess of Short-Term Dividend Rate over Long-Term Rate							
Issue	Current Rate	Issue Reset Spread	Next Reset Date	Expected Future Rate	Gross Difference per Annum	Number of payments before reset	Total Excess Payments
TRP.PR.A	$0.8165	192bp	2019-12-31	0.6425	0.1740	16	0.70
TRP.PR.B	$0.538	128bp	2020-6-30	0.4825	0.0555	18	0.25
TRP.PR.C	$0.56575	154bp	2021-1-30	0.5475	0.01825	20	0.09
TRP.PR.D	$1.00	238bp	2019-4-30	0.7575	0.2425	13	0.79
TRP.PR.E	$1.0625	235bp	2019-10-30	0.75	0.3125	15	1.17
TRP.PR.G	$0.95	296bp	2020-11-30	0.9025	0.0475	20	0.24
Note that the dividend rate of $0.56575 shown for TRP.PR.C formally becomes effective 2016-1-31, but the last quarterly dividend at the old rate has already been earned.							

When current prices have been adjusted by the indicated amounts, we derive Chart IV-23, which is slightly different from Chart IV-22, which used unadjusted prices. Thus, we conclude that while the approximation made in the 'normal' calculation is imprecise, it does not appear – in this case, at any rate – to be significant in an overall sense, although it will be noted that the errors are different in the more accurate calculation of Chart IV-23; most significantly, TRP.PR.E changes from 'very expensive' with unadjusted prices to 'quite expensive' after adjustment. It will be noted, however, that the change in rich/cheap analysis does not vary 1:1 with the size of the adjustment, as shown in Chart IV-24; the quality of the fit is much better with the adjusted prices.

TRP: Theoretical vs Expected Future Current Yield
January 8, 2016
Volatility 18%, Spread 385bp, GOC5 0.65%
Adjusted Prices

IV-24

Effect of Price Adjustment on Valuation of TRP FixedResets January 8, 2016

Given that the FixedReset structure largely removes the volatility of GOC5 from consideration, I would expect volatilities in the single digit range (compared to normal expectations of 15–20% for Straight Perpetuals).

Changes in Implied Volatility Over Time

Single series of preferred shares are too idiosyncratic for the purposes of long-term analysis, but as noted, the graphs which have been prepared showing YTW vs. Issue Reset Spread for the 'lesser credits' show the expected behavior (see Chart IV-14), with the advantage of being fairly comprehensive. Table IV-4 shows the changes in the slope of the regression line over time.

As Table IV-4 shows, the correlation between IRS and YTW (Chart FR-30) has not been very good throughout the downdraft of 2015, although we have seen a steady increase in both slope and correlation since the low of October 2015.

Table IV-4: Changes in Relationship Between YTW and IRS for "Pfd-2 Group" FixedResets Expected To Be Perpetul		
Date	Slope (x 10^5)	Correlation
2014-8-8	4.76	60%
2014-9-12	3.20	48%
2014-10-10	3.84	41%
2014-11-14	3.04	40%
2014-12-12	3.17	13%
2015-1-9	4.24	28%
2015-2-13	2.75	7%
2015-3-13	3.78	14%
2015-4-10	4.58	14%
2015-5-8	2.91	8%
2015-6-12	2.34	2%
2015-7-10	5.00	15%
2015-8-14	4.66	10%
2015-9-11	2.52	9%
2015-10-9	0.90	1%
2015-11-13	1.48	3%
2015-12-11	2.45	12%
2016-1-8	3.61	30%

A value of '10' in the Slope column would indicate a 1:1 correspondence between IRS and YTW; i.e., that increasing IRS by 1bp may be expected to result in an increased YTW of 1bp.

Understanding the "Spread"

A reader of PrefBlog asked[7] me to clarify what the "Spread" is.

The "Spread" is the spread which the company would have to offer – according to the data – in order to sell a true annuity, that is to say, a perpetual non-callable instrument paying the specified spread over five year Canadas.

This can get a little silly at times – in July, 2015, the indicated spread for RY was 88bp (and lower figures have been observed) which, given the contemporary GOC-5 yield of 1.00%, implied that they could have issued a true perpetual annuity at 1.88% (with resets at +88) … I don't think they could have sold such an issue! However, the unreasonableness of the answer implies that an inherent assumption in the calculation is wrong, which is useful information in itself. In the case of RY, the Implied Volatility was very high at 40%; unreasonably high, and I believe that in turn the reason for that is that the market is implicitly pricing in some directionality in the pricing, which contradicts the assumptions of the Black-Scholes model used in the calculation – an expected result for the NVCC non-compliant issues, but surprising for the compliant ones. The RY series has started to exhibit much more rational behaviour (although Implied Volatility is still very high) and investors profited from the change to the extent that they were expecting it.

Generally speaking, Implied Volatility rises with price … which is to say that when the entire series is priced in the range of 24.00 to 26.00 (say), the market will assume a high probability that the end price for everything will be about 25.00. However, when market yields change and everything is priced in the range of 21.00 to 23.00, the market gets depressed and assumes that nothing will ever get better and nothing will ever be called and the current price impairment for each instrument is permanent. This makes no sense, but that's the way it is! See the section *Effects of Proximity to Par Value* for more discussion.

The fact that we are seeing high levels of Implied Volatility even among series of FixedResets trading at low prices is suggestive that there is another factor at play; that the influence that causes instruments with different prices to trade at different yield is not just fear of a call (Implied Volatility) but there is also a certain amount of desire for the increased leverage against the GOC-5 yield and this desire is causing yield differentiation as well.

Another way of putting it is that the difference between the "Spread" and the actual Expected Future Current Yield is the price of the options; for example, with RY.PR.M the Expected Future Current Yield is currently 4.30% compared to the (0.65 + 3.25)% = 3.90% currently calculated for a non-callable perpetual annuity (using current figures from Chart IV-17), so the market is saying that RY's right to call RY.PR.M at $25.00 is worth 40bp of yield, each and every year.

Sometimes the market says silly things – in a perfect world, the Spread for any given issuer would be fairly constant (as it depends mainly on the credit quality of the issuer, although there will be a significant component due to the term premium and liquidity difference between a Five-Year Canada and a corporate perpetual annuity), while the Implied Volatility would vary somewhat with market conditions.

The Implications of High Volatility: Black-Scholes Option Pricing and N(d2)

We will also remember that in the Black-Scholes equation used to estimate these volatilities, the term N(d2) is the risk-adjusted probability of exercise, that is, they are the probabilities taking the expected return on the stock to be the risk-free rate.

To illustrate some results I consider reasonable, we can examine the figures for the BAM series of FixedResets (Chart IV-25) compared to MFC FixedResets, but given the extremely poor fit visible in IV-25, we will first repeat the exercise of adjusting the bid prices by the excess payments prior to reset, as was done with the TRP series above. This calculation is shown in Table IV-5 and the resultant Implied Volatility calculation illustrated in Chart IV-26.

Table IV-5: Price Reductions of BAM FixedResets To Reflect Excess of Short-Term Dividend Rate over Long-Term Rate							
Issue	Current Rate	Issue Reset Spread	Next Reset Date	Expected Future Rate	Gross Difference per Annum	Number of payments before reset	Total Excess Payments
BAM.PR.X	1.15	180	2017-6-30	0.6125	0.5375	6	0.81
BAM.PR.R	1.35	230	2016-6-30	0.7375	0.6125	2	0.31
BAM.PR.T	1.125	231	2017-3-31	0.74	0.385	5	0.48
BAM.PF.E	1.10	255	2020-3-31	0.80	0.30	17	1.28
BAM.PF.B	1.05	263	2019-3-31	0.82	0.23	13	0.75
BAM.PF.G	1.125	284	2020-6-30	0.8725	0.2525	18	1.14
BAM.PF.F	1.125	286	2019-9-30	0.8775	0.2475	15	0.93
BAM.PF.A	1.125	290	2018-9-30	0.8875	0.2375	11	0.65
BAM.PR.Z	1.20	296	2017-12-31	0.9025	0.2975	8	0.60
BAM.PF.H	1.25	417	2020-12-31	1.205	0.045	20	0.22
The calculation assumes that the future rate for BAM.PF.H is unaffected by the reset floor applicable to this issue.							

Although the fit using adjusted prices is better, as shown in Chart IV-26, it's still not very good! The rest of this section will discuss analytical results using unadjusted prices.
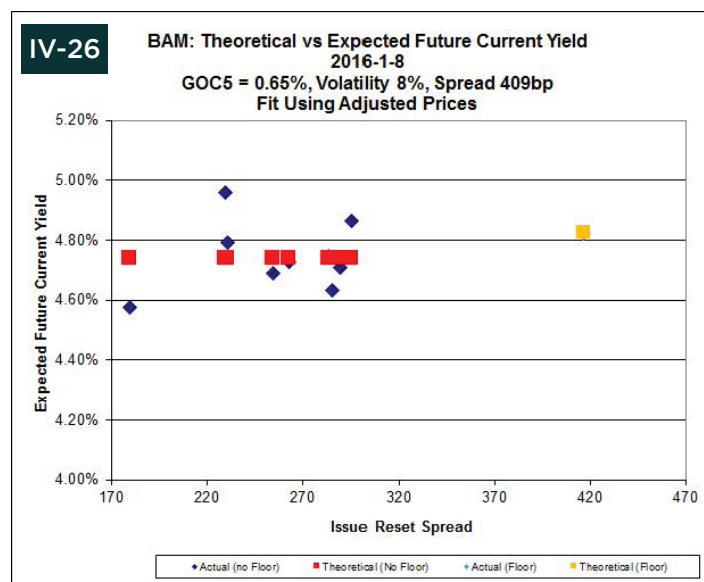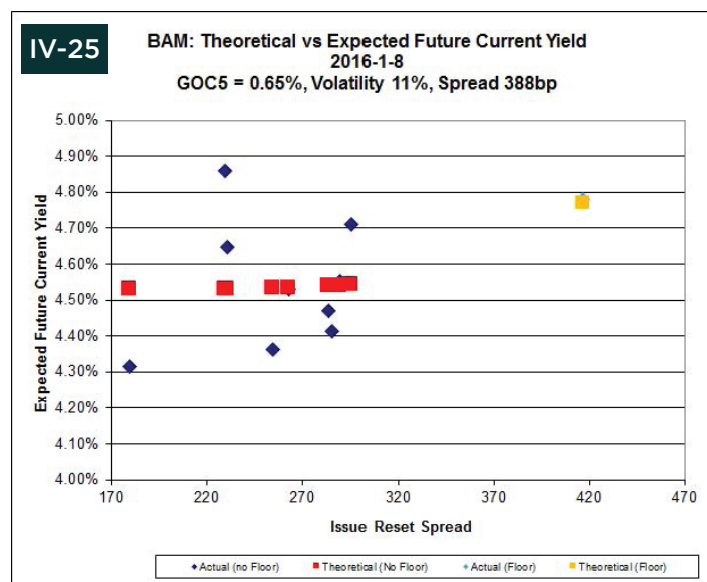
Although the fit of the BAM series is very poor, the Implied Volatility is reasonable and Table FR-13 will allow the reader to become more familiar with the probabilities of future option exercise under these conditions.

Table IV-6: Risk-Adjusted Option Exercise Probabilities for BAM FixedResets (Implied Volatility 2%, Spread 422bp)			
Ticker	Description	Price	$N(d_2)$ (Risk Adjusted Exercise Probability)
BAM.PR.X	4.60%+180	15.90	0.0%
BAM.PR.R	5.60%+230	15.18	0.0%
BAM.PR.T	4.50%+231	15.90	0.0%
BAM.PF.E	4.40%+255	18.34	0.2%
BAM.PF.B	4.20%+263	18.10	0.3%
BAM.PF.G	4.50%+284	19.52	1.0%
BAM.PF.F	4.50%+286	19.88	1.1%
BAM.PF.A	4.50%+290	19.50	1.3%
BAM.PR.Z	4.80%+296	19.16	1.6%
BAM.PF.H	5.00%+417M500	25.21	35.7%

The situation is very different when we repeat the exercise for the Manulife Financial (MFC) FixedResets (see Chart IV-18).

The fit is quite good – but we derive an Implied Volatility of 38% for MFC, which, as discussed, is unreasonably high. The implications of this difference with respect to exercise probability are shown in Table IV-7.

So, if we are to take the Implied Volatilities and related calculations at face value, we are required to believe that MFC.PR.G, with an Issue Reset Spread of 290bp, has a 35.3% chance of being called for redemption. This can be compared, for instance with BAM.PF.A, which also has a spread of 290bp but has an internally consistent exercise probability of 1.3%.

These results cannot be taken seriously. It is clear that at some point either the Implied Volatility for the MFC series will decline substantially, which implies that the slope of the Expected Future Current Yield (EFCY) vs. Issue Reset Spread relationship will become more shallow, which implies that the EFCY of the lower-spread issues will increase more than than the EFCY of the higher-spread issues, which allows us to conclude that the lower-spread MFC FixedResets are over-priced relative to their higher-spread siblings, or (and this is the alternative I favour!) that the NVCC rules will in fact be applied to MFC (which should lead to a steepening of the curve and an opposite conclusion regarding relative valuation).

Table IV-7: Risk-Adjusted Option Exercise Probabilities for MFC FixedResets (Implied Volatility 38%, Spread 266bp)			
Ticker	Description	Price	N(d_2) (Risk Adjusted Exercise Probability)
MFC.PR.F	4.20%+141	13.81	7.6%
MFC.PR.L	3.90%+216	18.01	21.4%
MFC.PR.K	3.80%+222	17.75	22.4%
MFC.PR.N	3.80%+230	18.61	24.2%
MFC.PR.M	3.90%+236	18.98	25.4%
MFC.PR.J	4.00%+261	19.45	30.2%
MFC.PR.I	4.40%+286	20.07	34.9%
MFC.PR.G	4.40%+290	19.34	35.3%
MFC.PR.H	4.60%+313	21.25	39.9%

Effects of Proximity to Par Value

The summer's volatility of higher-spread vs. lower-spread bank NVCC-compliant issues suggests the hypothesis that in a falling market, issues priced near par might experience smaller losses than issues priced significantly below par, irrespective of other characteristics. The causal mechanism for such an effect could be that retail investors (who continue to dominate the Canadian preferred share market) believe that:[8]

- Anything priced near par will always remain near par
- Anything priced significantly below par has something wrong with it

There is some support for this hypothesis in previous experience; for example, in October 2007 Rob Carrick of the Globe and Mail wrote an article regarding what he referred to as "Distressed Preferreds"[9] which he defined as *A distressed preferred trades below $20, which implies a 20-per-cent price decline, and it usually has a credit rating of less than pfd-3 (low) from DBRS Inc*.

This definition is clearly faulty since it makes no allowance for the yield of the instrument. It is entirely normal for a perfectly good long term issue to trade below 80% of par, if market yields have increased substantially above the coupon rate. However, the misconception is quite common amongst retail investors.

It should be noted that this effect may be easily confused with normal behaviour due to Implied Volatility, which will also provide a small cushion against rising yields, since an issue priced at par will have a value of the embedded call option that is large and negative; this value will move towards zero as yields rise and the price declines, which results in a lower "Effective Modified Duration" for issues; this can be exacerbated by a decline in Implied Volatility, which is generally observed as the price declines (although this makes no sense).

Investment Conclusions

Calculations of Implied Volatility for FixedResets shows promise as an analytical technique, but the market is not yet sophisticated enough to provide unambiguous data. However, this suggests that investors may achieve excess – albeit irregular – returns by selecting issues that are mispriced according to the theory.

---

[8]　This hypothesis was first described in http://prefblog.com/?p=28441 and christened the "Par Always, Shit Forever Hypothesis" on Financial Webring Forum (http://www.financialwisdomforum.org/forum/viewtopic.php?f=33&t=113976&start=550#p549392)

[9]　This article is no longer available on-line; it was discussed at http://prefblog.com/?p=1326